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An investigation into some of the effects of cross coupling between the longitudinal and lateral equations of motion of a fighter type aircraft at subsonic speed.

Emanski, John Joseph

Monterey, California: U.S. Naval Postgraduate School

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AN INVESTIGATION INTO SOME  
OF THE EFFECTS OF CROSS COUPLING  
BETWEEN THE LONGITUDINAL AND LATERAL EQUATIONS  
OF MOTION OF A FIGHTER TYPE AIRCRAFT AT SUBSONIC SPEED

A Thesis Submitted to the  
Faculty of the Graduate School of the  
University of Minnesota

by

John Joseph Emanski, Jr.  
Commander, United States Navy

In Partial Fulfillment of the Requirements  
for the Degree of Master of Science  
in Aeronautical Engineering

August, 1955



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## SUMMARY

Modern aircraft design trends have been toward aircraft with long, dense fuselages, smaller aerodynamic surfaces, swept wings and higher wing loadings. These aircraft are required to fly at high altitudes. Flight data from such aircraft have indicated that the linearized dynamic equations do not always predict the dynamic behavior accurately. In addition, certain cross-coupling effects were developing between the lateral and longitudinal motion of the aircraft.

The effect of the trend toward concentrating the mass in the fuselage was investigated by making an engineering approximation of the change in the moments of inertia caused by this change in mass distribution. Four configurations were considered, each one approaching nearer the limit of a missile or body of revolution. Since the lower lift curve slopes and smaller wing areas of the swept-wing aircraft sometimes require flight at very high angles of attack, the effect of inclination of the principal longitudinal axis to the flight path was investigated. The effect of sweep was approximated by varying the effective dihedral parameter,  $C_{\ell\beta}$ . In addition the effect of varying  $C_{\eta\beta}$ , the "weather-cock-stability" parameter, was studied since long fuselages affect this parameter adversely.

The investigation was made using a Reeves Electronic Analogue Computer (REAC) by setting up a mathematical model of a representative military-type airplane flying





at a Mach number of 0.6 at sea level and at 40,000 feet. The mathematical model was based on equations of motion which were extended to include products of the perturbation velocity terms. The effect of changing the mass distribution, the inclination of the longitudinal principal axis, and variation of  $C_{n_\beta}$  and  $C_{l_\beta}$  was investigated by changing the mathematical model as required by the parameter change considered and obtaining the REAC response to a fixed or standard step-function control input.

The results indicated that the lateral motion coupled into the longitudinal motion and that this effect was increased as the fuselage became longer and denser. Altitude aggravated this coupling as it did any trend toward instability caused by a change in the other parameters considered. The rudder caused greater coupling than the aileron. A positive inclination of the principal longitudinal axis caused an increase in dynamic stability and a decrease in coupling while a negative inclination had the opposite effect. A decrease in  $C_{n_\beta}$  produced a marked trend toward instability and an increase in coupling while a decrease in  $C_{l_\beta}$  had the opposite effect.

The investigation required by this thesis was carried out in the Computer Center of the University of Minnesota.





## INTRODUCTION

Fighter aircraft of recent design show a trend toward long fuselages where most of the mass is concentrated, smaller aerodynamic surfaces, high rates of roll, and increased use of sweepback. Flight data from such aircraft indicate that the linear equations of motion, which result after several simplifying assumptions, do not always predict accurately the dynamic behavior. In addition, these same design trends have required more emphasis to be placed on the consideration of the dynamic stability of an aircraft in the design stage. In earlier practice, dynamic stability usually took care of itself after the static stability requirements were met.

The effect on coupling and dynamic stability caused by the parameters principally affected by these design trends was investigated by means of a mathematical model entered on the Reeves Electronic Analogue Computer (REAC). An attempt was made to use a representative or typical aircraft of the fighter type. The mathematical model was based on equations of motion which were extended to include the products of the velocity change usually assumed to be zero.

The mass distribution change was investigated by considering four configurations each more nearly approaching a missile from the standpoint of its moment of inertia properties. The effect of the inclination of the principal longitudinal axes was considered at



inclination angles of zero, plus ten, and minus ten degrees. The effect of  $C_{n_\beta}$  and  $C_{l_\beta}$  was studied by changing the mathematical model as required by a change in these parameters over a range of values. Altitude effects were considered by calculating the dimensional derivatives for 40,000 feet, changing the mathematical model accordingly, and running the program as it was done for the sea level case.

The effect of a parameter change was observed by obtaining successive REAC responses to a standard step function control input. Only the parameter investigated was changed so that any variation in the time history response was attributed to the parameter change. The data was presented by superimposing these successive time histories. The trend of the effect of the parameter change considered could then be easily observed.





## DEVELOPMENT OF THE PROBLEM

The classical approach to airplane dynamics is outlined here in a very brief manner. The airplane was considered to be a rigid body. Six equations were required to define its motion; one force equation and one moment equation for each axis. These equations included the force and moment components due to aerodynamic and mass reactions together with the force of gravity and thrust effects. Second order effects were neglected and the equations simplified to a degree which permitted hand solution.

Appendix I develops the equations of motion used in this investigation. The classical approach has been used in general but certain departures have been made. In Appendix II the equations developed in Appendix I have been simplified by assuming that the airplane initially is in straight and level, steady, gliding flight. The thrust effects were allowed to go to zero since they are not critically essential in a dynamic stability study.

Derivative terms of order higher than one in the series expansion of the expressions for the forces and moments were neglected. The effect of these terms is known to be secondary, data was available only for the first derivative, and inclusion of these terms, even if available, would require more components than the REAC's provided. A second assumption generally made is that



the product of the perturbation velocities is essentially zero. This yields two independent sets of three linear equations. This assumption was not made. The equations simplified in Appendix II are presented below:

$$\begin{aligned}
 (\text{II-7}) \quad \dot{u} &= X_u u + X_q q + X_w w + X_{\dot{w}} \dot{w} + v r - w q - g \theta \\
 \dot{w} &= Z_u u + Z_q q + Z_w w + Z_{\delta_e} \delta_e + q u - p v \\
 \ddot{\theta} &= \dot{q} = M_u u + M_q q + M_w w + M_{\delta_e} \delta_e + M_{\dot{w}} \dot{w} \\
 &\quad - p r \left( \frac{I_x - I_z}{I_y} \right) + r^2 \frac{I_{yz}}{I_y} - p^2 \frac{I_{xz}}{I_y} \\
 \dot{v} &= Y_r r + Y_v v + Y_p p + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r + w p - r u + g \theta \\
 \ddot{\phi} &= \dot{p} = L_r r + L_v v + L_p p + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r \\
 &\quad + r \frac{I_{xz}}{I_x} - q r \left( \frac{I_z - I_y}{I_x} \right) + p q \frac{I_{xz}}{I_x} \\
 \ddot{\psi} &= \dot{r} = N_r r + N_v v + N_p p + N_{\delta_r} \delta_r + N_{\delta_a} \delta_a \\
 &\quad + \dot{p} \frac{I_{xz}}{I_z} + p q \left( \frac{I_y - I_x}{I_z} \right) - q r \frac{I_{xz}}{I_z}
 \end{aligned}$$

The axes used and the sign convention observed is presented in Table I and illustrated in Fig. 8 which heads Table I.

Ref. 2 points out that motion of the longitudinal symmetric type does not induce motion of the lateral asymmetric type. An examination of the equation set presented above indicates that there are no terms which would couple symmetric motion into the asymmetric motion if the asymmetric motion were initially zero.

Duncan in Ref. 2 also points out that when deviations are so small that their squares and products can be neglected, there will be no coupling of the lateral motion into the symmetric motion. When the deviations





are not small longitudinal motion will persist alone, but a purely lateral motion will, in general, induce longitudinal motion. The motion will couple into the forward velocity through the term  $v_r$ ; the pitching velocity through the terms  $p_r \left( \frac{I_x - I_z}{I_y} \right)$ ,  $n^2 \frac{I_{xz}}{I_y}$  and  $p^2 \frac{I_{xz}}{I_y}$ ; and into the normal velocity, through the term  $p_v$ . It can be observed that the coupling terms are modified generally by moments and products of inertia.

The study of the effect of the trend in mass distribution toward a long, dense fuselage was made by considering four cases starting with a hypothetical fighter airplane of conventional type. The characteristics of this basic airplane are presented in Table II. The fuselage was considered to represent 60 per cent of the airplane's mass. It was taken to have a maximum width of five feet and to have an eight-to-one slenderness ratio. The pitching moment of inertia calculated using these assumptions and the approximation that, for purposes of making an engineering estimate, the fuselage can be considered a long, slender cylinder resulted in a calculated value of the pitching moment of inertia which was very near the  $I_y$  of the airplane. As the mass became completely concentrated in the fuselage it was assumed that the slenderness ratio would increase to twelve to one, the fuselage width would remain the same, and that the pitching and yawing moments of inertia could be calculated using the long cylinder approximation. In this final form, the aircraft would



be a missile with  $I_z$  equal to  $I_y$  and  $I_x$  very small.

Fig. 10 illustrates the engineering approximation that was made of the variation in moments of inertia as the fuselage became longer and denser. The mass distribution changes were made while the aerodynamic damping terms, control power terms, mass and other effects were considered constant. This would, of course, be unrealistic but it has the advantage of studying the effect of a change in one parameter in the "partial derivative" sense.

The product of inertia term resulted largely from the fact that stability axes were used. When the

$x$  -axis is so oriented in an airplane in the steady flight condition that it is parallel to the relative wind, the Eulerian axes are referred to as stability axes. NACA TR 589 indicated that the  $I_{xz}$  terms were unimportant for conventional aircraft, however, Ref. 7 showed that the  $I_{xz}$  term may have a pronounced effect on the lateral stability of some high speed airplanes. It has already been noted that the  $I_{xz}$  term is involved in the coupling of the lateral and longitudinal motion.

If the longitudinal axes are inclined at an angle  $\eta$  to the flight path, the moments of inertia about the flight path axis and the axis normal to the flight path are:

$$\begin{aligned} I_x &= I_{x_0} \cos^2 \eta + I_{z_0} \sin^2 \eta \\ I_z &= I_{z_0} \cos^2 \eta + I_{x_0} \sin^2 \eta \\ I_{xz} &= -(I_{z_0} - I_{x_0}) \sin \eta \cos \eta \end{aligned}$$





The angle  $\eta$  largely results from the wing incidence angle and the angle of attack.

The trend toward swept wings and long fuselages makes  $C_{l_\beta}$  and  $C_{n_\beta}$  increasingly important. Long fuselages adversely affect  $C_{n_\beta}$ , the "weather-cock" parameter. Swept wings have very large values of effective dihedral thus affecting,  $C_{l_\beta}$ , the effective dihedral parameter. The effect of a change in these parameters on coupling and on dynamic stability was investigated. The change in the static derivative characteristics was selected for study since general flying characteristics are influenced primarily by the static stability parameters according to Ref. 6.

The coefficients such as  $L_v$  are dimensional stability derivatives. They are direct functions of airframe geometry, mass, mass distribution and flight conditions; in addition to being direct functions of the non-dimensional stability derivatives such as,  $C_{l_\beta}$ . These basic non-dimensional stability derivatives are, in turn, implicit functions of airframe geometry, mass, mass distribution and flight conditions. The values of the non-dimensional stability derivatives for the basic airplane appear as Table II. The dimensional stability derivatives are defined in Tables III and IV. The values of the dimensional derivatives were calculated and appear as Table V. Refs. 1, 2, 3, 4, 7, 8, and 9 discuss these derivatives, their characteristics, and how they are determined. They are assumed to be



constant for a certain range of applicability and flight conditions.

The dimensional stability derivatives can be used directly as numerical coefficients in the dynamic equations of the airframe based on real time. Thus, stability derivatives in this form are used in setting up the mathematical model for the analogue computer. The use of this form of derivative permits the extension of the non-dimensional derivatives to other flight conditions with only secondary error. This was done to obtain the mathematical model of the airplane at 40,000 feet. Certain values of the non-dimensional derivatives which were specifically applicable to flight at this altitude were used while others were extended by the change in the values of  $\rho$  and  $U$ .

Phillips in Ref. 4 considered the effect of steady rolling on longitudinal and directional stability. The equations which he used can be arrived at from the equations developed in Appendix I by applying the assumptions that he made in order to arrive at an equation set which could be handled analytically. He retained the product terms which couple the longitudinal and lateral motions. He found that rapid rates of roll caused instability if the directional and longitudinal stabilities are different when the rolling frequency exceeds the lower of the pitching and yawing natural frequencies of the non-rolling airplane. The instability existed only while the air-





plane was rolling. In airplanes of short span and high fuselage density flying at high altitudes, this instability may cause dangerous attitude changes. Further, he found that when an airplane rolls about an axis which is not aligned with its longitudinal principal axis, forces were introduced which tended to swing the fuselage out of line with the flight path. These are the inertial forces usually neglected when the linearized equations are used.

In Ref. 4, Phillips made many assumptions in an effort to obtain equations which were capable of hand solution yet would predict the effect on coupling of steady rolling. In the investigation conducted incident to this thesis, the effects of the three motions, roll, pitch, and yaw, on coupling as produced by step-function control inputs were investigated. As many influencing factors were retained as the REAC's components would permit.

Solutions of the equations of motion used in this thesis would require the simultaneous solution of six differential equations, three of second order, made non-linear by product terms such as  $q\dot{r}$  and  $\dot{p}r$ . The labor involved would have been prohibitive. The equations were fitted to two REAC's and four servo-multipliers. The solutions were recorded simultaneously on two four-channel Sanborn recorders.



## EQUIPMENT AND PROCEDURE

The equipment used to conduct the investigation consisted of two REAC's, four servo-multiplier units, two four channel Sanborn recorders, and a one-shot multi-vibrator time switch. The entire computing system is illustrated in the photograph presented as Fig. 2. The Sanborn recorder is shown in Fig. 3, the lateral motion and control function computer in Fig. 4, the longitudinal motion computer in Fig. 5, and the time switch in Fig. 6.

Each of the four servo-mechanisms drove three multiplying potentiometers. These multipliers provided  $\frac{1}{100}$  of the product of the inputs. This restriction made it necessary to build up the perturbation velocities to ten times their actual value in order to obtain a simple product such as  $\mu r$ . The additional amplification required by the servo-multipliers made computer balance very delicate and this was a source of difficulty.

The Sanborn recorder made time history records of four quantities simultaneously. The imprint was made by a heated pen on sensitized paper. The point did not touch the paper, thus eliminating the drag usually associated with a recording pen.

The time switch was designed to provide an accurately timed step-function as a standard input. Basically, the switch is a one-shot multi-vibrator which fires just once when the operating button is pressed. This allows





current to flow in the coil of a relay in the plate circuit of the "normally-off" tube. The relay contacts send plus or minus 100 volts through the aileron, rudder, or elevator function switch, whichever is closed. The time that the relay remains closed is governed by the RC time constant controlled by the variable potentiometer. A range of from approximately 0.5 to 2.5 seconds was available. The electrical schematic is presented as Fig. 7. A photograph of the "bread-board" model appears as Fig. 6. The contacts of the relay carrying plus or minus 100 volts would appear electrically just ahead of the function switches in the REAC solution schematic which appears as Fig. 9.

The six equations of motion were put in computer form in Appendix II. These equations are II-13 through II-18. A computer schematic for the simultaneous solution of these equation was laid-out and appears as Fig. 9. From the schematic, patchboards were wired for the lateral motion and control computer and the longitudinal motion computer. This division of the problem was based on the wiring scheme which would use the fewest amplifiers. The apparent symmetry was incidental. The numerical values of the quantities entering the problem were translated into potentiometer settings. The values recorded were  $u$  ,  $10\theta$  ,  $w$  ,



and  $10\dot{\phi}$  on one recorder;  $\psi$ ,  $10\phi$ ,  $10\dot{\psi}$ , and  $\dot{\theta}$  on the other recorder. The plus and minus 100 volts was placed on one side of the timer relay contacts. The other side of the relay contacts was connected to each of the control function switches, plus or minus, whichever was required by the schematic.

The graph representing the estimated mass distribution changes as the fuselage becomes longer and more dense is presented in Fig. 10. Values of the moments of inertia of three configurations were picked off this graph. New stability derivatives were calculated as necessary for these configurations. All of the moment derivatives such as  $L_r$  were affected. These calculated values were translated into potentiometer settings and appear as Tables VII and VIII.

The configurations investigated were called Case I, II, III and IV. The lateral computer was referred to as the "blue" computer and the longitudinal computer as the "red" computer. These are occasionally referred to in the tables using the abbreviations B and R.

The computers were balanced and the recorders calibrated. The patchboards were then installed on the computers. A rudder, elevator, or aileron control movement was then selected and the recorder tape set in motion at the desired speed. Pushing the time switch



The first of these is the fact that the  
 number of cases of the disease has  
 been increasing steadily since the  
 first case was reported in 1918.  
 This is due to the fact that the  
 disease is now more common than  
 it was in 1918. The second fact  
 is that the disease is now more  
 common in the United States than  
 it was in 1918. The third fact  
 is that the disease is now more  
 common in the world than it was  
 in 1918. The fourth fact is that  
 the disease is now more common  
 in the United States than it was  
 in 1918. The fifth fact is that  
 the disease is now more common  
 in the world than it was in 1918.  
 The sixth fact is that the disease  
 is now more common in the United  
 States than it was in 1918. The  
 seventh fact is that the disease  
 is now more common in the world  
 than it was in 1918. The eighth  
 fact is that the disease is now  
 more common in the United States  
 than it was in 1918. The ninth  
 fact is that the disease is now  
 more common in the world than  
 it was in 1918. The tenth fact  
 is that the disease is now more  
 common in the United States than  
 it was in 1918.

button sent an accurately timed standard step-function through the control switch selected. The solutions of the equations of motion in response to a step-function input were recorded as time histories of the variables on eight recorder channels simultaneously. The next case was then entered on the potentiometers and the solutions obtained as before. These solutions are presented as Figs. 12A, 12B, 13A, 13B, 14A, and 14B.

The dimensional stability derivatives were recalculated using data for 40,000 feet and the required angle of attack increase obtained from Table II. Where specific non-dimensional derivatives were not available for this altitude, those for low altitude were extended by using the values of  $\rho$  and  $U$  at 40,000 feet in the formulae of Tables III and IV. These stability derivatives and other factors of the problem which changed at 40,000 feet were translated into potentiometer settings and run off as before. The settings appear in Tables IX and X. The results for Cases I and IV are presented in Figs. 15A, 15B, 16A, 16B, 17A and 17B. It should be pointed out that the airplane which was marginal in dynamic directional stability at sea-level was unstable at 40,000 feet. In order to "fly" even marginally at this altitude the  $N_r$  potentiometer was set to 0.0115 and the  $L_r$  potentiometer to zero. This had the effect of increasing the "weather-cock" stability parameter about six times and reducing the effective dihedral

The first part of the paper is devoted to a general discussion of the problem of the origin of life. It is shown that the problem is not only a scientific one, but also a philosophical one. The scientific aspect of the problem is concerned with the question of how life arose from non-life. The philosophical aspect is concerned with the question of whether life is a necessary part of the universe or whether it is a mere accident.

The second part of the paper is devoted to a discussion of the various theories of the origin of life. It is shown that there are three main theories: the theory of spontaneous generation, the theory of panspermia, and the theory of abiogenesis. Each of these theories is discussed in detail, and the evidence for and against each is presented.

The third part of the paper is devoted to a discussion of the evidence for the origin of life. It is shown that there is a great deal of evidence in favor of the theory of abiogenesis. This evidence includes the discovery of the fossil record, the discovery of the chemical evolution of life, and the discovery of the genetic code.

The fourth part of the paper is devoted to a discussion of the implications of the origin of life. It is shown that the origin of life has important implications for our understanding of the universe and for our understanding of ourselves. It is also shown that the origin of life has important implications for the search for life on other planets.

parameter to zero. The computer procedure was as before. It should be noted that solutions were obtained only for Cases I and IV modified as described above.

The effect of inclination of the longitudinal principal axis to the flight path was investigated by calculating values of moments and products of inertia for inclination angles  $\eta$  of plus ten, zero, and minus ten degrees. The equations below obtained from Ref. 6 were used:

$$I_x = I_{x_0} \cos^2 \eta + I_{z_0} \sin^2 \eta$$

$$I_z = I_{z_0} \cos^2 \eta + I_{x_0} \sin^2 \eta$$

$$I_{xz} = -(I_{z_0} - I_{x_0}) \sin \eta \cos \eta$$

It was found that only the  $I_{xz}$  value varied significantly. New values of the coefficients containing  $I_{xz}$  were calculated and translated into potentiometer settings. These are presented in Table XI. Several sign changes were required. These were accomplished by using inverters as necessary. The computer was operated as before using Case I at low altitude.

The  $C_{n\beta}$  and  $C_{l\beta}$  programs were accomplished by changing these values over the range considered, recalculating the affected stability derivatives, and translating them into potentiometer settings. This was done for Case I at sea level and at 40,000 feet. These values are presented in Tables XII and XIII. The computer operation was similar to the other programs.





## RESULTS AND DISCUSSION

Time histories of the variables  $u$ ,  $w$ ,  $\theta$ ,  $v$ ,  $\phi$ ,  $\dot{\theta}$ ,  $\dot{\phi}$ , and  $\dot{\psi}$  were recorded as the airplane responded to a step function control input for each change in a parameter being investigated. To study the trend of the effect of the parameter changes, the curves were superimposed, each curve being drawn with a characteristic line. These composite solution curves are presented as Figs. 12A through 23B. The standard step function time plot appears as Fig. 11. It should be noted that the A and B parts of the figures each represent one of the four channels which were simultaneously recorded. Where separate lines for each parameter change can not be observed, it should be assumed that the curve lies behind the previous curve, or curves, and that no significant change was caused by the new value of the parameter.

The mass distribution program is presented in the Figs. 12A, 12B, 13A, 13B, 14A, and 14B. The elevator response curves show no significant changes caused by the changed moments of inertia. The elevator set-up the phugoid motion which remained essentially constant. The short period motion was virtually dead-beat damped. The amplitude of  $\dot{\theta}$  in Case IV is reduced since the aircraft was more sluggish to pitch change due to the increased pitching moment of inertia. A slight dutch-roll was induced. This was not anticipated since the equations indicated that there would be no coupling





from the symmetric motion into the lateral motion if there was no initial lateral motion. A possible explanation might be that there was some unbalance in the servo-multipliers or jitter in the lateral motion computer due to small line current variations. The same effect might be observed in flight tests caused by slightly unsymmetrical control application or small trim unbalance. Spiral instability was set-up presumably by the same mechanism which caused the slight dutch-roll to start.

The cross-coupling set-up by the aileron and rudder motion can be observed in the  $\omega$  plot. The coupling caused by the more assymmetric motion induced by the rudder step is several times that of the aileron cross-coupling. The aileron's cross-coupling decreased as the values of  $I_y$  and  $I_z$  were increased. This was unexpected but can possibly be explained by the fact that two opposing effects were taking place. The coupling increased but the airplane became more sluggish since the pitch and yaw moments of inertia were increased. This sluggishness can be observed in the progressive decrease in amplitude and frequency of  $u$  and  $v$ . The large decrease in roll moment of inertia had very little effect on  $\phi$  and  $\dot{\phi}$ .

The altitude effects on coupling and dynamic stability are presented in Figs. 15A, 15B, 16A, 16B, 17A, and 17B. Many of the observations made in the



sea-level program apply here. The short period oscillation damping decreased increasing the period of this motion. The aileron coupling was larger for Case IV, that is, as the fuselage became long and dense. The same effect was observed in the coupling caused by the rudder. This time there was little amplitude difference between the aileron and rudder coupling. The sluggishness caused by increased  $I_y$  and  $I_z$ , together with decreased aerodynamic damping, made the swings progressively larger in amplitude and period. The roll angle did not significantly change.

In the program which considered the effect of changing the inclination of the longitudinal principal axis to the flight path, the  $I_{xz}$  program essentially, only the rudder and aileron step input response was investigated. As  $\eta$  became more positive, dynamic stability increased; more negative, dynamic stability decreased. At an  $\eta$  of minus ten degrees, dynamic instability developed. From the coupling standpoint, the same effect was observed. There is less coupling when  $\eta$  becomes more positive; more when  $\eta$  becomes more negative. The rudder curves show these effects more clearly than the aileron curves.

The effects caused by varying the "weather-cock" parameter,  $C_{n\beta}$ , is presented in Figs. 20A, 20B, 21A and 21B. Only the aileron and rudder forcing functions were considered. The curves for two values of  $C_{n\beta}$



at high altitude were superimposed on the sea-level curves. At both altitudes similar effects were observed. Coupling increased as  $C_{n\beta}$  decreased. The dynamic lateral stability decreased as  $C_{n\beta}$  decreased. This was to be expected, of course, since decreased tail arm, or tail area, lead toward lateral instability. As the angular velocity changes increased with this instability the coupling increased.

The curves for the  $C_{l\beta}$  program are presented in Figs. 22A, 22B, 23A and 23B. The entire sea-level program was presented but only the curve for a  $C_{l\beta}$  of 0.20 at 40,000 feet was added. As  $C_{l\beta}$  was increased dynamic, lateral instability quickly developed along with increasing coupling. The effect was more apparent in the motion caused by the rudder forcing function. The reduced damping at altitude hastened the development of instability and the coupling increased. The curve for a  $C_{l\beta}$  of 0.20 was marginally stable at sea level but unstable at 40,000 feet.

The parameters  $C_{n\beta}$  and  $C_{l\beta}$  had very significant effects on dynamic stability. The effective dihedral parameter,  $C_{l\beta}$ , was very critical. This would appear to make analysis of the dynamics of the airframe very important in the design stage of swept-wing and delta-wing aircraft. The REAC provides a powerful tool for such an analysis.





The use of a standard step-function input provided a good means for studying the effect of a parameter change. However, this method was inadequate from the standpoint of bringing out any instabilities caused by cross-coupling of the type investigated in Ref. 4.

It should be pointed out that the dynamic behavior observed was that of one airplane, or variation of one airplane. Although effort was made to make the mathematical model investigated "typical", it would not be reasonable to suggest that all airplanes behave in the manner of the airplane investigated. On the other hand, the procedures used in the analysis were straight-forward and it is felt that the dynamic behavior observed might be considered reasonably typical.



## CONCLUSIONS AND RECOMMENDATIONS

1. Mass distribution changes had no significant effect on the motion of the aircraft in the longitudinal plane.
2. The motion of the aircraft in the symmetrical plane coupled into the lateral motion in the form of a slight dutch roll and also triggered the spiral instability mode. However, this is believed to have been due to slight assymetry in the computing system.
3. The assymmetrical motion of the airplane coupled into the motion in the symmetrical plane and is most significant as a change in  $\omega$ . This would be equivalent to a change in the angle of attack.
4. The coupling effect increased as the fuselage became longer and more dense.
5. The assymmetrical motion caused by the rudder had a greater coupling effect than the motion caused by the aileron.
6. At high altitudes both the coupling and the trend toward instability increased.
7. At a positive inclination of the principal longitudinal axis to the flight path, there was less coupling and increased dynamic stability. A negative inclination angle had the opposite effect.



8. A decrease in the "weather-cock" stability parameter,  $C_{n\beta}$ , resulted in a decrease in dynamic stability and an increase in coupling.
9. An increase in the effective dihedral parameter,  $C_{l\beta}$ , resulted in a decrease in dynamic stability and an increase in coupling.
10. The standard step-function input which supplied the forcing function provided a means for studying the effect of a parameter change on the coupling and the dynamic stability of the airplane. However, it does not appear adaptable to the investigation of instabilities which may develop from the coupling caused by rapid rolling or yawing. An investigation might do this by using a programmed input which would reverse before the REAC limits were reached.

The dynamic behavior observed in this investigation was that of the mathematical model of one airplane or variation of one airplane. Effort was made to use a mathematical model which could be considered reasonably "typical". However, it would not be reasonable to suggest that all airplanes would respond similarly.





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SYMBOLS

$U_0$	initial airspeed, feet per second
$\rho$	mass density of air, slugs per cubic foot
$b$	wing span, feet
$S$	wind area, square feet
$W$	weight of airplane, pounds
$m$	mass, slugs
$g$	acceleration of gravity, feet per second per second
$AR$	aspect ratio
$M$	Mach number
$C_t$	wing chord tip, feet
$C_r$	wing chord root, feet
$I_{x_0}$	moment of inertia about principal longitudinal axis, slug-feet <sup>2</sup>
$I_{y_0}$	moment of inertia about principal tranverse axis, slug-feet <sup>2</sup>
$I_{z_0}$	moment of inertia about principal normal axis, slug-feet <sup>2</sup>
$I_{x_z}$	moment of inertia about flight path axis, slug-feet <sup>2</sup>
$I_y$	moment of inertia about the pitching axis, slug-feet <sup>2</sup>
$I_z$	moment of inertia about the yawing axis, slug-feet <sup>2</sup>
$I_{xz}$	product of inertia with respect to the flight path axis and axis normal to flight path, slug-feet <sup>2</sup>





- F force, pounds
- h moment of momentum, (slug-ft)-ft per second
- $C_L$  lift coefficient,  $-\frac{\text{Lift force}}{q S}$
- $C_L$  rolling moment coefficient,  $\frac{\text{Rolling moment}}{q S b}$
- $C_Y$  lateral-force coefficient,  $\frac{\text{lateral force}}{q S}$
- $C_m$  aerodynamic pitching moment coefficient about the c.g.
- $C_D$  drag coefficient,  $\frac{\text{Drag force}}{q S}$
- $\phi$  angle of bank, radians
- $\theta$  angle of pitch, radians
- $\psi$  angle of yaw, radians
- $\delta$  control deflection, radians
- $\beta$  angle of sideslip, radians
- $\gamma$  flight path angle; angle between relative wind  
and the horizontal, radians
- $\eta$  angle between the longitudinal axis of the fuse-  
lage and the relative wind, radians
- $\xi$  angle between the thrust line and the longitudinal  
axis of the fuselage, radians
- $z_j$  perpendicular distance from c.g. to the thrust  
line, feet
- $\zeta$  fraction of critical damping
- $\omega$  frequency, radians/sec.
- U initial forward velocity in direction of positive  
axis (feet per second)
- V initial side velocity (feet per second)
- W initial normal velocity (feet per second)



- u forward velocity change after disturbance (feet per second)
- v side velocity change after disturbance (feet per second)
- w normal velocity change after disturbance (feet per second)
- P initial rolling angular velocity (radians per second)
- Q initial pitching angular velocity (radians per second)
- R initial yawing angular velocity (radians per second)
- $p, \dot{\phi}$  rolling angular velocity increment, radians per second
- $q, \dot{\theta}$  pitching angular velocity increment, radians per second
- $r, \dot{\psi}$  yawing angular velocity increment, radians per second
- $C_{D_u}$  the change in drag coefficient with varying forward velocity
- $C_{D_\alpha}$  the change in drag coefficient with varying angle of attack, per radian
- $C_{y_p}$  rate of change of lateral-force coefficient with rolling-angular-velocity factor, per radian
- $C_{y_r}$  rate of change of lateral-force coefficient with yawing-angular-velocity factor, per radian
- $C_{y_\beta}$  lateral force derivative, rate of change of lateral force coefficient with angle of sideslip, per radian



- $C_{Y_{\delta r}}$  the change in side force coefficient with variation in rudder deflection, per radian
- $C_{Y_{\delta a}}$  the change in side force coefficient with aileron deflection, per radian
- $C_{L_u}$  change in lift coefficient with variation in forward velocity, angle of attack and attitude remaining constant
- $C_{L_q}$  change in lift coefficient with varying pitching velocity, no change of airplane angle of attack as a whole, per radian
- $C_{L_\alpha}$  the change in lift coefficient with varying angle of attack, "lift curve slope", per radian
- $C_{L_{\delta e}}$  change in lift coefficient with changes in elevator deflection, per radian
- $C_{l_r}$  rate of change of rolling moment coefficient with yawing angular velocity factor, per radian
- $C_{l_\beta}$  effective-dihedral derivative, rate of change of rolling moment coefficient with angle of side-slip, per radian
- $C_{l_p}$  damping in roll derivative, rate of change of rolling moment coefficient with rolling-angular-velocity factor, per radian
- $C_{l_{\delta r}}$  change in rolling moment coefficient with variation in rudder deflection, per radian
- $C_{l_{\delta a}}$  aileron effectiveness on aileron power, the change in rolling moment coefficient with change in aileron deflection, per radian





- $C_{m_\alpha}$  longitudinal static stability derivative, change in pitching moment coefficient with varying angle of attack, per radian
- $C_{m_\alpha'}$  the change in pitching moment coefficient with variation in rate of change of angle of attack, per radian
- $C_{m_q}$  pitch damping derivative, change in pitching moment coefficient with varying pitch velocity, per radian
- $C_{m_{\delta_e}}$  elevator effectiveness or elevator power, the change in pitching moment coefficient with change in elevator deflection, per radian
- $C_{n_\beta}$  directional stability derivative, weather-cock stability, rate of change of yawing-moment coefficient with angle of sideslip  $\left( \frac{\partial C_n}{\partial \beta} \right)$  per radian
- $C_{n_r}$  damping in yaw derivative, rate of change of yawing moment with yawing angular velocity factor, per radian  $\left( \frac{\partial C_n}{\partial \frac{rb}{2U_0}} \right)$
- $C_{n_p}$  rate of change of yawing moment coefficient with rolling angular velocity factor, per radian  $\left( \frac{\partial C_n}{\partial \frac{pb}{2U_0}} \right)$
- $C_{n_{\delta_r}}$  rudder effectiveness or rudder power, the change in yawing moment coefficient with variation in rudder deflection, per radian
- $C_{n_{\delta_a}}$  change in yawing moment coefficient with change in aileron deflection, per radian
- $X_u$  change in fore and aft force with change in forward velocity, per second
- $X_q$  change in fore and aft force, change in pitching velocity, feet per second-radian





- $X_w$  change in fore and aft force with change in  
normal velocity, per second
- $X_{\dot{w}}$  change in fore and aft force with rate of change  
in normal velocity, per foot
- $X_r$  change in side force with change in yawing velocity,  
per radian
- $Y_v$  change in side force with change in side velocity,  
per second-radian
- $Y_p$  change in side force with change in rolling velocity,  
per radian
- $Y_{\delta_a}$  change in side force with aileron deflection,  
per second-radian
- $Y_{\delta_r}$  change in side force with rudder deflection, per  
second-radian
- $Z_u$  change in normal force with varying forward velocity,  
per second
- $Z_q$  change in normal force with varying pitching velocity,  
feet per second-radian
- $Z_w$  change in normal force with varying vertical  
velocity, per second
- $Z_{\delta_e}$  change in normal force with elevator deflection,  
feet per second<sup>2</sup>-radian
- $L_r$  change in rolling moment with varying yawing velocity,  
per second-radian
- $L_v$  change in rolling moment with varying side velocity,  
per second-feet



$L_p$  change in rolling moment with varying rolling velocity, per second-radian

$L_{\delta_r}$  change in rolling moment with rudder deflection, per second<sup>2</sup>-radian

$L_{\delta_a}$  change in rolling moment with aileron deflection, per second<sup>2</sup>-radian

$M_u$  change in pitching moment with varying forward velocity, per second-feet

$M_w$  change in pitching moment with varying vertical velocity, per second-feet

$M_{\dot{w}}$  change in pitching moment with rate of change of vertical velocity, per foot

$M_q$  change in pitching moment with varying pitching velocity, per second-radian

$M_{\delta_e}$  change in pitching velocity with elevator deflection, per second<sup>2</sup>-radian

$N_v$  change in yawing moment with varying side velocity, per second-feet

$N_r$  change in yawing moment with varying yawing velocity, per second-radian

$N_p$  change in yawing moment with varying rolling velocity, per second-radian

$N_{\delta_r}$  change in yawing moment with rudder deflection, per second<sup>2</sup>-radian

$N_{\delta_a}$  change in yawing moment with aileron deflection, per second<sup>2</sup>-radian

$N_\beta$  change in yawing moment with side-slip angle, per second<sup>2</sup>-radian



- ( )<sub>o</sub> refers to initial steady flight condition
- ( )<sub>x</sub> pertains to the fore and aft axis
- ( )<sub>y</sub> pertains to the transverse axis
- ( )<sub>z</sub> pertains to the normal axis
- ( )<sub>e</sub> elevator
- ( )<sub>a</sub> aileron
- ( )<sub>r</sub> rudder





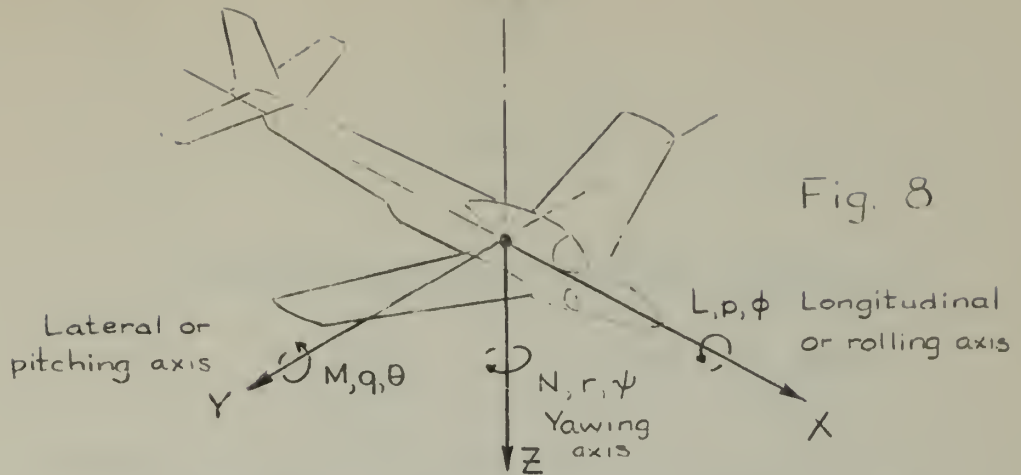


TABLE I NOTATION FOR AXES SYSTEM:

Axis Designation	Longitudinal	Lateral	Yawing
Symbol	X	Y	Z
Force (parallel to axis) symbol	X	Y	Z
Sign convention	Forward +	To right +	Down +
Linear Velocity	U	V	W
Change in linear velocity	u	v	w
Sign Convention	Forward +	To right +	Down +
Moment about axis	Rolling moment	Pitching moment	Yawing moment
Symbol	L	M	N
Coefficient	$C_L$	$C_m$	$C_n$
Formula	$L = \frac{1}{2} \rho V^2 S b C_L$	$M = \frac{1}{2} \rho V^2 S c C_m$	$N = \frac{1}{2} \rho V^2 S b C_n$
Sign convention	Right roll + ( $Y \rightarrow Z$ )	Nose up + ( $Z \rightarrow X$ )	Nose right + ( $X \rightarrow Y$ )
Moment of Inertia	$I_x$	$I_y$	$I_z$
Angular Displacement	Roll angle	Pitch angle	Yaw angle
Symbol	$\phi$	$\theta$	$\psi$
Sign convention	Right roll +	Nose up +	Nose right +
Angular Velocity	Rolling velocity	Pitching velocity	Yawing velocity
Symbol	P	Q	R
Change in angular velocity	$p = \dot{\phi}$	$q = \dot{\theta}$	$r = \dot{\psi}$
Sign convention	Right roll +	Nose up +	Nose right +



Table II

CHARACTERISTICS OF THE BASIC AIRPLANE

M = 0.6

Altitude = Sea level

W	30,000 lbs.	$C_D$	.024 at $C_L = .4$	$C_{Yp}$	-.02 at $C_L = 0$
m	932 slugs	$C_L$	.1 at s.l.	$C_{Yp}$	+.04 at $C_L = .6$
$U_o$	670 ft/sec	$C_L$	.56 at 40,000'	$C_{Y\delta_r}$	.062
$\rho_{sl}$	.00238 $\frac{\text{slugs}}{\text{ft}^3}$	$C_{D_u}$	0	$C_{n_r}$	-.03
S	630 ft <sup>2</sup>	$C_{D\alpha}$	.05 at $C_L = .1$	$C_{n_p}$	.008 at $C_L = 0$
b	56 ft.	$C_{D\alpha}$	.21 at $C_L = .4$	$C_{n_p}$	-.024 at $C_L = .6$
$C_t$	7.5 ft.	$C_{L_u}$	.02 at $C_L = .1$	$C_{n\delta_r}$	-.036
$C_r$	15 ft	$C_{L\alpha}$	+ 5.0	$C_{n\delta_a}$	.008 at $C_L = 0$
AR	5	$C_{L\delta_i}$	1.4	$C_{n\delta_a}$	-.012 at $C_L = .6$
$I_x$	53,000 slug-ft <sup>2</sup>	$C_{L_g}$	3.8	$C_{l_b}$	-.10 heavy wing loading
$I_y$	80,000 slug-ft <sup>2</sup>	$C_{L\delta_e}$	-.17	$C_{l_r}$	.012 at $C_L = 0$
$I_z$	124,000 slug-ft <sup>2</sup>	$C_{m_u}$	-.002 at $C_L = .1$	$C_{l_r}$	.11 at $C_L = .6$
$I_{xz}$	4000 slug-ft <sup>2</sup>	$C_{m\alpha}$	-.24 c.g. at 25% MAC	$C_{l_p}$	-.44 heavy wing loading
$I_{x_o}$	52,500 slug-ft <sup>2</sup>	$C_{m\alpha}$	- 3.1	$C_{l\delta_r}$	.008 at $C_L = 0$
$I_{y_o}$	80,000 slug-ft <sup>2</sup>	$C_{m\eta}$	- 10.6	$C_{l\delta_r}$	.003 at $C_L = .5$
$I_{z_o}$	124,500 slug-ft <sup>2</sup>	$C_{m\delta_e}$	.81	$C_{l\delta_a}$	.176
$\eta$	- 3.25°	$C_{Y\beta}$	-.32 at $C_L = 0$	$C_{n\beta}$	.055
$C_D$	.014 at $C_L = 0.1$	$C_{Y_r}$	.12	$C_{m.c.g.}$	0



Table III

LONGITUDINAL STABILITY DERIVATIVES			
$X_u$	$\frac{\rho U S}{m}(-C_{Du}-C_D)$	$X_q \doteq$	0
$Z_u$	$\frac{\rho U S}{m}(-C_{Lu}-C_L)$	$Z_q$	$-\frac{\rho U S c}{4m} C_{Lq}$
$M_u$	$\frac{\rho U S c}{I_x}(C_{mu}+C_m)$	$M_q$	$\frac{\rho U S c^2}{4I_y} C_{mq}$
$T_u$	$\frac{\rho U S}{m}(C_{Tu}+G)$	$T_{\delta rpm}$	$\frac{30\rho U S c}{m} C_{T_{\delta rpm}}$
$X_w$	$\frac{\rho U S}{2m}(C_L-C_{D\alpha})$	$X_{\delta e}$	$-\frac{\rho U^2 S}{2m} C_{D_{\delta e}}$
$Z_w$	$\frac{\rho U S}{2m}(C_{L\alpha}-C_D)$	$Z_{\delta e}$	$-\frac{\rho U^2 S}{2m} C_{L_{\delta e}}$
$M_w$	$\frac{\rho U S c}{2I_y} C_{m\alpha}$	$M_{\delta e}$	$\frac{\rho U^2 S c}{2I_y} C_{m_{\delta e}}$
$X_{\dot{w}} \doteq$	0	$X_{\delta f}$	$-\frac{\rho U^2 S}{2m} C_{D_{\delta f}}$
$Z_{\dot{w}}$	$-\frac{\rho S c}{4m} C_{L\dot{\alpha}}$	$Z_{\delta f}$	$-\frac{\rho U^2 S c}{2m} C_{L_{\delta f}}$
$M_{\dot{w}}$	$\frac{\rho S c^2}{4I_y} C_{m\dot{\alpha}}$	$M_{\delta f}$	$\frac{\rho U^2 S c}{2I_y} C_{m_{\delta f}}$

Table IV

LATERAL STABILITY DERIVATIVES			
$Y_r$	$\frac{\rho U S}{2m} C_{Y\beta}$	$Y_r$	$\frac{\rho U S b}{4m} C_{Yr}$
$L_r$	$\frac{\rho U S b}{2I_x} C_{L\beta}$	$L_r$	$\frac{\rho U S b^2}{4I_x} C_{Lr}$
$L_\beta$	$U_0 L_r$	$N_r$	$\frac{\rho U S b^2}{4I_z} C_{Nr}$
$N_r$	$\frac{\rho U S b}{2I_z} C_{N\beta}$	$Y_{\delta r}$	$\frac{\rho U^2 S}{2m} C_{Y_{\delta r}}$
$N_\beta$	$U_0 N_r$	$L_{\delta r}$	$\frac{\rho U^2 S b}{2I_x} C_{L_{\delta r}}$
$Y_p$	$\frac{\rho U S b}{4m} C_{Yp}$	$N_{\delta r}$	$\frac{\rho U^2 S b}{2I_z} C_{N_{\delta r}}$
$L_p$	$\frac{\rho U S b^2}{4I_x} C_{Lp}$	$Y_{\delta a}$	$\frac{\rho U^2 S}{2m} C_{Y_{\delta a}}$
$N_p$	$\frac{\rho U S b^2}{4I_z} C_{Np}$	$L_{\delta a}$	$\frac{\rho U^2 S b}{2I_x} C_{L_{\delta a}}$
		$N_{\delta a}$	$\frac{\rho U^2 S b}{2I_z} C_{N_{\delta a}}$





Table V

DIMENSIONAL DERIVATIVES OF THE BASIC AIRPLANE			
M = 0.6 ALTITUDE, SEA LEVEL			
$X_u, \frac{1}{\text{sec}}$	-0.0151	$L_v, \frac{1}{\text{sec-ft}}$	-0.054
$X_q, \frac{\text{ft}}{\text{sec-rad}}$	0	$L_p, \frac{1}{\text{sec-rad}}$	-6.53
$X_w, \frac{1}{\text{sec}}$	+0.0269	$L_{\delta_r}, \frac{1}{\text{sec}^2\text{-rad}}$	+2.84
$X_{\dot{w}}, \frac{1}{\text{ft}}$	0	$L_{\delta_a}, \frac{1}{\text{sec}^2\text{-rad}}$	+62.5
$Y_r, \frac{1}{\text{rad}}$	+1.81	$M_u, \frac{1}{\text{sec-ft}}$	-0.00282
$Y_v, \frac{1}{\text{sec-rad}}$	-0.173	$M_w, \frac{1}{\text{sec-ft}}$	-0.034
$Y_p, \frac{1}{\text{rad}}$	-0.302	$M_q, \frac{1}{\text{sec-rad}}$	-16.86
$Y_{\delta_a}, \frac{1}{\text{sec-rad}}$	0	$M_{\delta_c}, \frac{1}{\text{sec}^2\text{-rad}}$	+76.0
$Y_{\delta_r}, \frac{1}{\text{sec-rad}}$	+22.4	$M_{\dot{w}}, \frac{1}{\text{ft}}$	-0.00184
$Z_u, \frac{1}{\text{sec}}$	-0.130	$N_v, \frac{1}{\text{sec-ft}}$	+0.0125
$Z_q, \frac{\text{ft}}{\text{sec-rad}}$	+11.55	$N_r, \frac{1}{\text{sec-rad}}$	-0.190
$Z_w, \frac{1}{\text{sec}}$	-2.70	$N_p, \frac{1}{\text{sec-rad}}$	+0.0508
$Z_{\delta_c}, \frac{\text{ft}}{\text{sec}^2\text{-rad}}$	+61.3	$N_{\delta_a}, \frac{1}{\text{sec}^2\text{-rad}}$	+1.21
$L_r, \frac{1}{\text{sec-rad}}$	+17.85	$N_{\delta_r}, \frac{1}{\text{sec}^2\text{-rad}}$	-5.46



-36-  
Table VI

POTENTIOMETER SETTINGS FOR LATERAL AND CONTROL  
COMPUTER MASS DISTRIBUTION PROGRAM (Sea Level)

POTENTIOMETER	PARAMETER AFFECTED	CASE I	CASE II	CASE III	CASE IV
1	$\frac{1}{4} \frac{I_{xz}}{I_y}$	.013	.0058	.0028	.001
2	$\frac{I_x - I_z}{I_y}$	.890	.943	.970	.982
3	$\frac{1}{40} M g$	.435	.260	.185	.143
4	$\frac{1}{2} g$	.819	.819	.819	.819
5	$X_u$	.015	.015	.015	.015
6	$Z_u$	.130	.130	.130	.130
7	$\frac{1}{1000} U_0$	.674	.674	.674	.674
8	$\frac{1}{40} Z_g$	.297	.297	.297	.297
9	$\frac{1}{2} Z_w$	.692	.692	.692	.692
10	$X_w$	.027	.027	.027	.027
11	$M_w$	.034	.0203	.0145	.0112
12	$Z_{\delta_e} (2.5^\circ)$	.027	.027	.027	.027
13	$\frac{1}{100} M_{\delta_e} (2.5^\circ)$	.034	.0203	.0145	.0112
14	$\frac{1}{100} N_{\delta_a} (5^\circ)$	.0011	.0008	.0006	.0005
15	$\frac{1}{100} N_{\delta_R} (5^\circ)$	.0047	.0035	.0028	.0023
16	$\frac{1}{10} (N_{\delta_a} + N_{\delta_R})$	.103	.103	.103	.103
17	$Y_{\delta_R} (5^\circ)$	.020	.020	.020	.020
18	$\frac{1}{100} L_{\delta_R} (5^\circ)$	.0025	.0032	.0045	.0077
19	$\frac{1}{100} L_{\delta_a} (5^\circ)$	.055	.071	.100	.172
20	$\frac{1}{4} (Y_{\delta_R} + w \dot{\theta})$	.256	.256	.256	.256
21	$\frac{1}{4} (L_{\delta_a} + L_{\delta_R})$	.256	.256	.256	.256



POTENTIOMETER SETTINGS FOR LONGITUDINAL COMPUTER  
MASS DISTRIBUTION PROGRAM - SEA LEVEL

POTENTIOMETER SETTING	PARAMETER AFFECTED	CASE I	CASE II	CASE III	CASE IV
1	$N_V$	.013	.010	.008	.006
2	$\frac{N_R}{10}$	.019	.0141	.0011	.0092
3	$\frac{1}{4} \left( \frac{I_4 - I_1}{I_E} \right)$	.055	.1405	.190	.224
4	$\frac{I_{XE}}{I_E}$	.033	.0184	.0097	.004
5	$\frac{N_P}{10}$	.005	.0037	.0029	.0024
6	$\frac{1}{4} \frac{I_{XE}}{I_E}$	.008	.0045	.0024	.001
7	$\frac{I_E - I_4}{I_X}$	.859	.833	.821	.733
8	$\frac{1}{4} \frac{I_{XE}}{I_X}$	.019	.0184	.0174	.0148
9	$\frac{I_{XE}}{I_X}$	.076	.0737	.0695	.0593
10	$\frac{L_P}{100}$	.067	.087	.122	.209
11	$\frac{1}{10} L_X$	.018	.0232	.033	.0562
12	$\frac{1}{1000} U_0$	.711	.711	.711	.711
13	$\frac{1}{40} g$	.819	.819	.819	.819
14	$\frac{1}{10} Y_R$	.182	.182	.182	.182
15	$\frac{1}{10} Y_P$	.030	.030	.030	.030
16	$Y_V$	.174	.174	.174	.174
17	$L_V$	.054	.0696	.0987	.168





Table VIII

MOMENTS AND PRODUCTS OF INERTIA

FOR CASES I, II, III AND IV

Case	I	II	III	IV
$I_x$ (slug - ft <sup>2</sup> )	53,000	41,000	29,000	17,000
$I_y$ (slug - ft <sup>2</sup> )	80,000	134,000	188,000	243,000
$I_z$ (slug - ft <sup>2</sup> )	124,000	167,000	211,000	255,000
$I_{xz}$ (slug - ft <sup>2</sup> )	4,000	3,000	2,000	1,000



POTENTIOMETER SETTINGS  
FOR LATERAL AND CONTROL COMPUTER  
MASS DISTRIBUTION PROGRAM (40,000 FEET)

POTENTIOMETER	CASE I	CASE II	CASE III	CASE IV
1	.013	.0058	.0028	.0011
2	.890	.943	.970	.982
3	.0925	.0553	.0384	.0308
4	.819	.819	.819	.819
5	.0070	.0070	.0070	.0070
6	.134	.134	.134	.134
7	.583	.583	.583	.583
8	.0632	.0632	.0632	.0632
9	.1474	.1474	.1474	.1474
10	.0299	.0299	.0299	.0299
11	.0073	.0043	.0031	.0024
12	.0099	.0099	.0099	.0099
13	.0128	.0074	.0053	.00408
14	.0005	.0004	.0003	.0003
15	.0025	.0018	.0014	.0012
16	.103	.103	.103	.103
17	.0018	.0018	.0018	.0018
18	.0001	.0002	.0003	.0006
19	.005	.007	.0092	.0143
20	.256	.256	.256	.256
21	.256	.256	.256	.256

Note:  $\delta_e = 5^\circ$   
 $\delta_n = \delta_a = 2.5^\circ$



-42-  
Table X

POTENTIOMETER SETTINGS  
FOR LONGITUDINAL COMPUTER

MASS DISTRIBUTION PROGRAM 10,000 FEET

POTENT- IOMETER	CASE I	CASE II	CASE III	CASE IV
1	.0115	(.0019)	(.0016)	.0056
2	.0041	.003	.0024	.0020
3	.055	.1405	.190	.224
4	.033	.0184	.0097	.004
5	.00328	.0024	.0019	.0016
6	.008	.0045	.0024	.001
7	.859	.833	.821	.733
8	.019	.0184	.0174	.0148
9	.076	.0737	.0695	.0593
10	.0139	.0181	.0253	.0433
11	.0348	.0448	.0638	.11087
12	.583	.583	.583	.583
13	.819	.819	.819	.819
14	.0386	.0386	.0386	.0386
15	.0013	.0013	.0013	.0013
16	.037	.037	.037	.037
17	0	(.0148)	(.0210)	0

Note: Cross-hatching indicate  
a sign change.





Table XI

POTENTIOMETER SETTINGS

FOR  $I_{xz}$  PROGRAM

$\eta$	$\frac{I_{xz}}{I_z}$ 4R	$\frac{1}{4} \frac{I_{xz}}{I_z}$ 6R	$\frac{1}{4} \frac{I_{xz}}{I_x}$ 8R	$\frac{I_{xz}}{I_x}$ 9R	$\frac{1}{4} \frac{I_{xz}}{I_y}$ 1B	$\frac{1}{4} \frac{I_y - I_x}{I_z}$ 3R	$\frac{I_z - I_y}{I_z}$ 7R	$\frac{I_x - I_z}{2B}$
-10°	.105	.0255	.074	.227	.050	.067	.813	.850
0°	0	0	0	0	0	.0553	.859	.902
+10°	<del>.105</del>	<del>.0255</del>	<del>.074</del>	<del>.227</del>	<del>.050</del>	<del>.067</del>	<del>.813</del>	<del>.850</del>

Cross hatch = sign change

Table XII

$C_{\eta\beta}$  PROGRAM

$C_{\eta\beta}$	$N_r$ (1R) SEA LEVEL	$N_r$ (1R) 40,000 FT
+ .0	0	0
+ .055	.013	
+ .10	.0236	.0073
+ .15	.0354	
+ .30	.0708	.0155

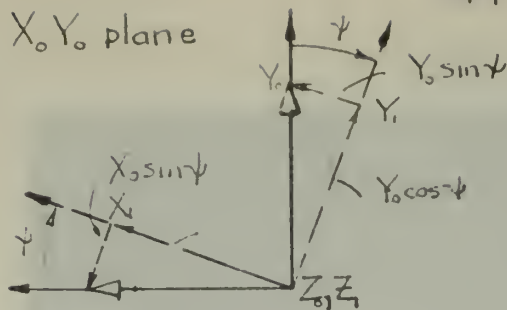
Table XIII

$C_{\ell\beta}$  PROGRAM

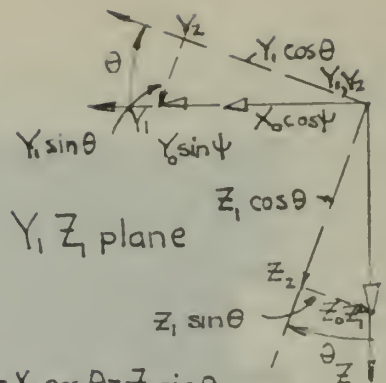
$C_{\ell\beta}$	$L_r$ (17R)	$L_r$ (17R)
0	0	0
.10	.054	.0115
.20	.108	.0230
.30	.162	
.40	.216	



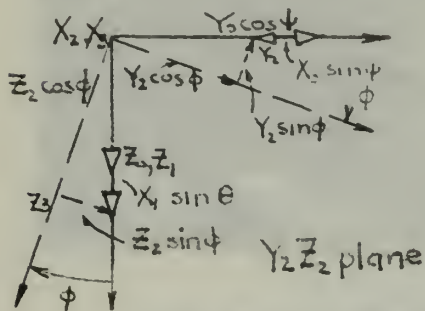
$X_0 Y_0$  plane



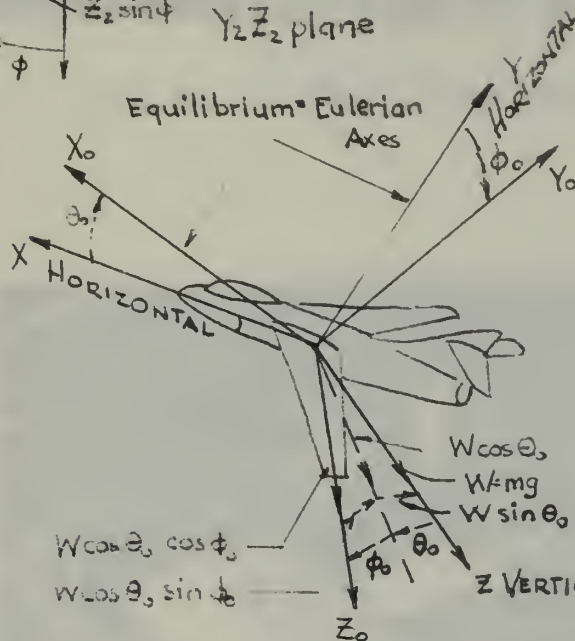
$$\begin{aligned} X_1 &= X_0 \cos \psi + Y_0 \sin \psi \\ Y_1 &= Y_0 \cos \psi - X_0 \sin \psi \\ Z_1 &= Z_0 \end{aligned}$$



$$\begin{aligned} X_2 &= X_1 \cos \theta - Z_1 \sin \theta \\ Y_2 &= Y_1 \\ Z_2 &= Z_1 \cos \theta + X_1 \sin \theta \end{aligned}$$



$$\begin{aligned} X_3 &= X_2 \\ Y_3 &= Y_2 \cos \phi + Z_2 \sin \phi \\ Z_3 &= Z_2 \cos \phi - Y_2 \sin \phi \end{aligned}$$



$$\begin{aligned} X_0 &= -W \sin \theta_0 \\ Y_0 &= W \cos \theta_0 \sin \phi_0 \\ Z_0 &= W \cos \theta_0 \cos \phi_0 \end{aligned}$$

Components of gravity acting on an aircraft in steady flight with initial angles  $\theta_0$  and  $\phi_0$

$$X_3 = (-W \sin \theta_0) \cos \theta \cos \psi + (W \cos \theta_0 \sin \phi_0) \cos \theta \sin \psi - (W \cos \theta_0 \cos \phi_0) \sin \theta$$

$$\begin{aligned} Y_3 &= (-W \sin \theta_0) (\cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi) + (W \cos \theta_0 \sin \phi_0) (\cos \psi \cos \theta \\ &\quad + \sin \psi \sin \theta \sin \phi) + (W \cos \theta_0 \cos \phi_0) (\cos \theta \sin \phi) \end{aligned}$$

$$\begin{aligned} Z_3 &= (-W \sin \theta_0) (\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) + (W \cos \theta_0 \sin \phi_0) (\sin \psi \sin \theta \cos \phi \\ &\quad - \cos \psi \sin \phi) + (W \cos \theta_0 \cos \phi_0) (\cos \theta \cos \phi) \end{aligned}$$

Fig. 1 DETERMINATION OF THE COMPONENTS OF GRAVITY ACTING ALONG DISTURBED EULERIAN AXES



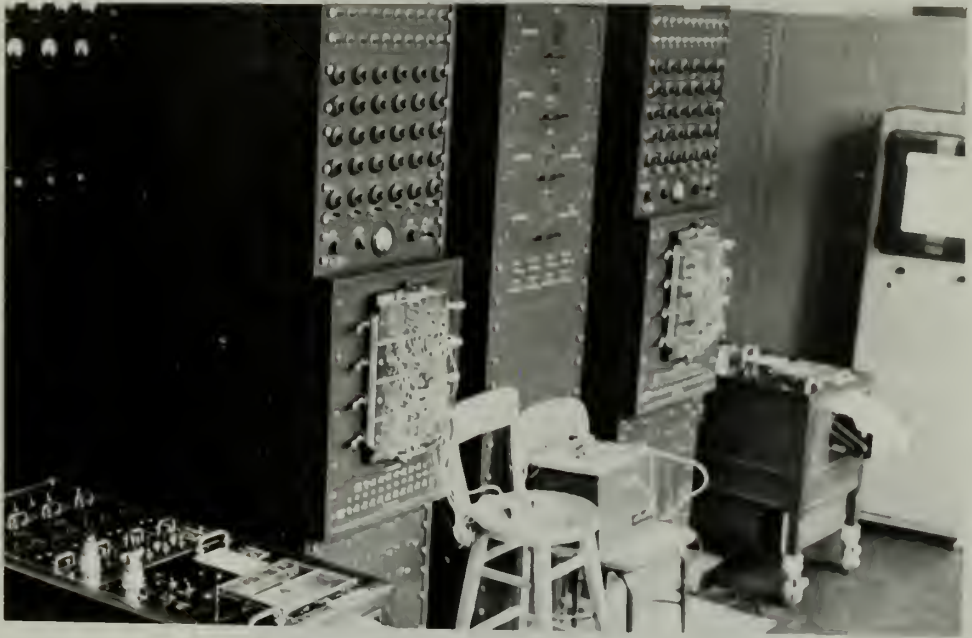


Fig. 2 THE COMPUTING SYSTEM

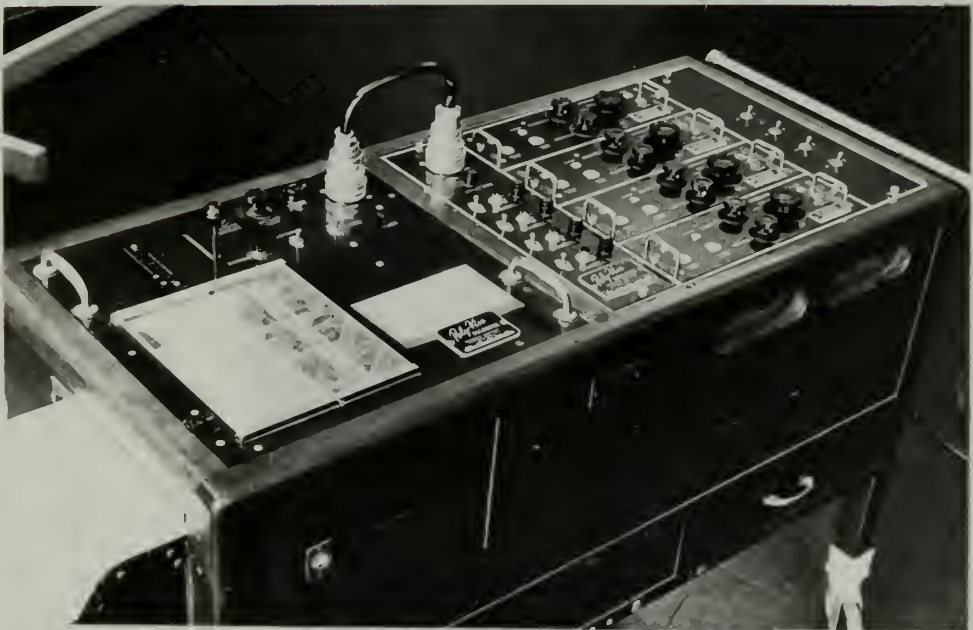


Fig. 3 THE SANBORN FOUR-CHANNEL RECORDER





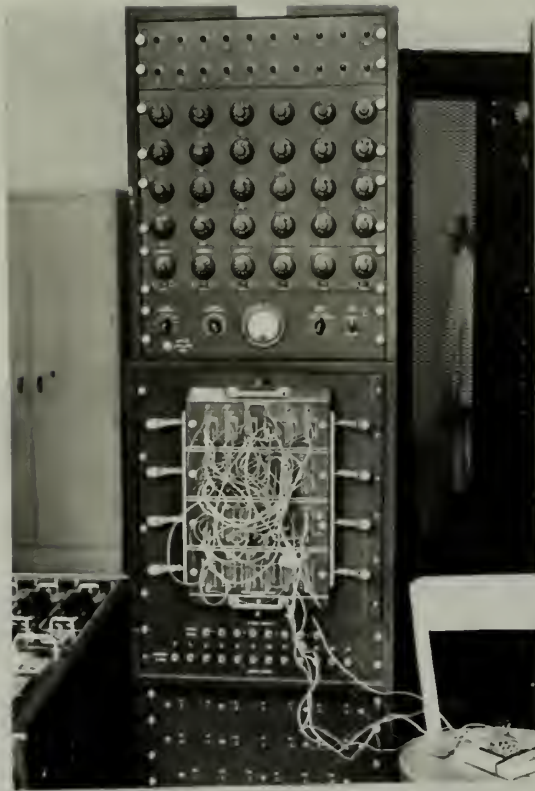


Fig. 4 THE LATERAL MOTION AND  
CONTROL FUNCTION COMPUTER



Fig. 5 THE LONGITUDINAL MOTION  
COMPUTER



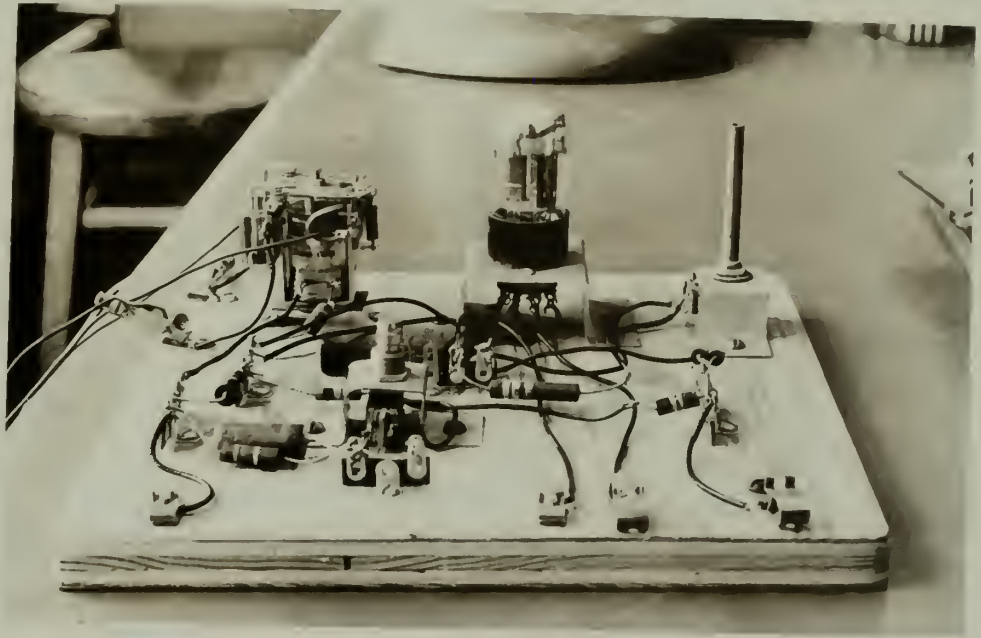
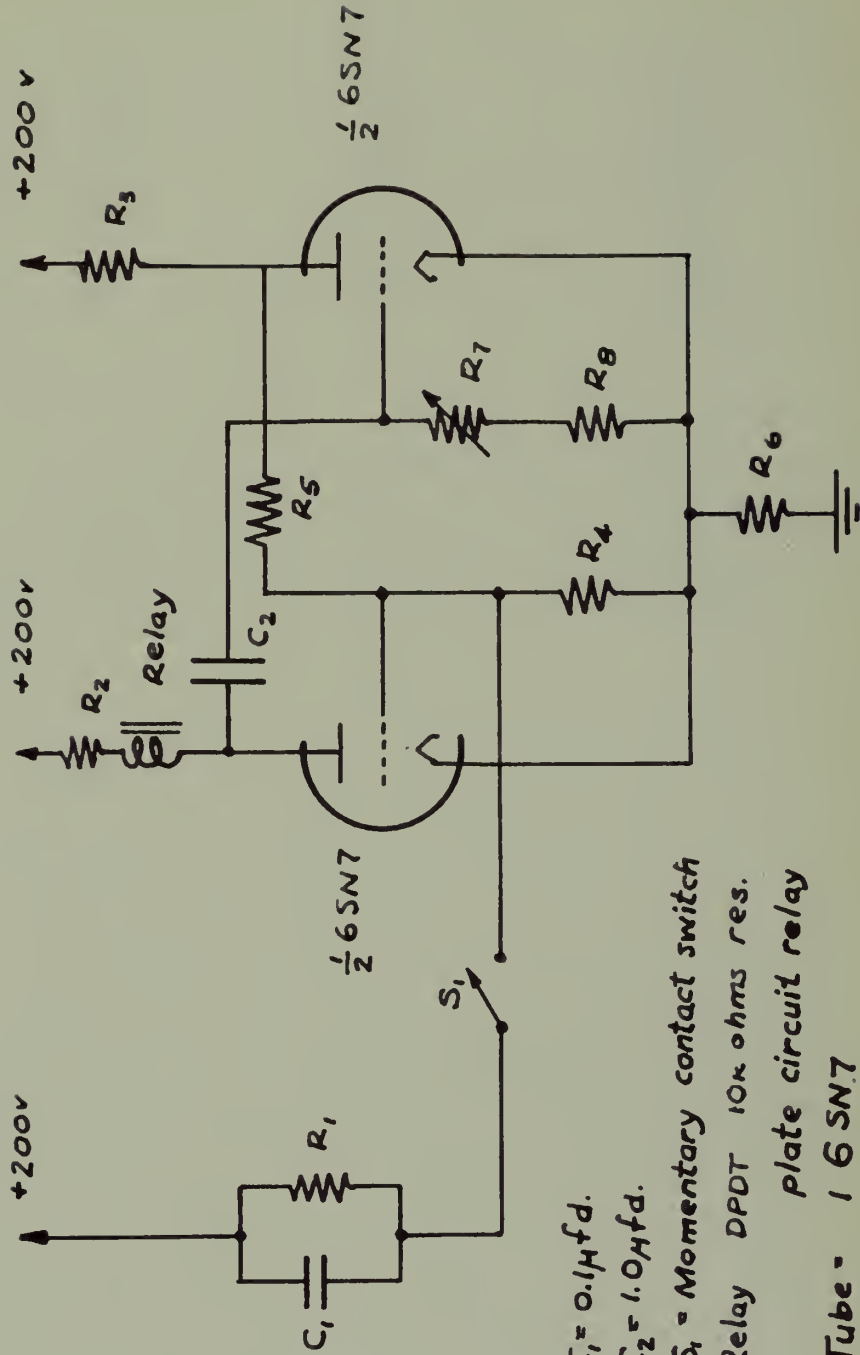


Fig. 6 THE ONE-SHOT MULTIVIBRATOR TIME SWITCH





- $R_1 = 10^6 \text{ ohms}$   
 $R_2 = 10\kappa "$   
 $R_3 = 20\kappa "$   
 $R_4 = 150\kappa "$   
 $R_5 = 300\kappa "$   
 $R_6 = 1/\kappa "$   
 $R_7 = 750\kappa "$   
 $R_8 = 250\kappa "$
- $C_1 = 0.1\mu\text{fd.}$   
 $C_2 = 1.0\mu\text{fd.}$   
 $S_1 = \text{Momentary contact switch}$   
 Relay DPDT 10 $\kappa$  ohms res.  
 plate circuit relay  
 Tube = 1 6SN7  
 $R_8 = 750\kappa "$  linear pot.

Fig. 7 ONE SHOT MULTIVIBRATOR TIME SWITCH





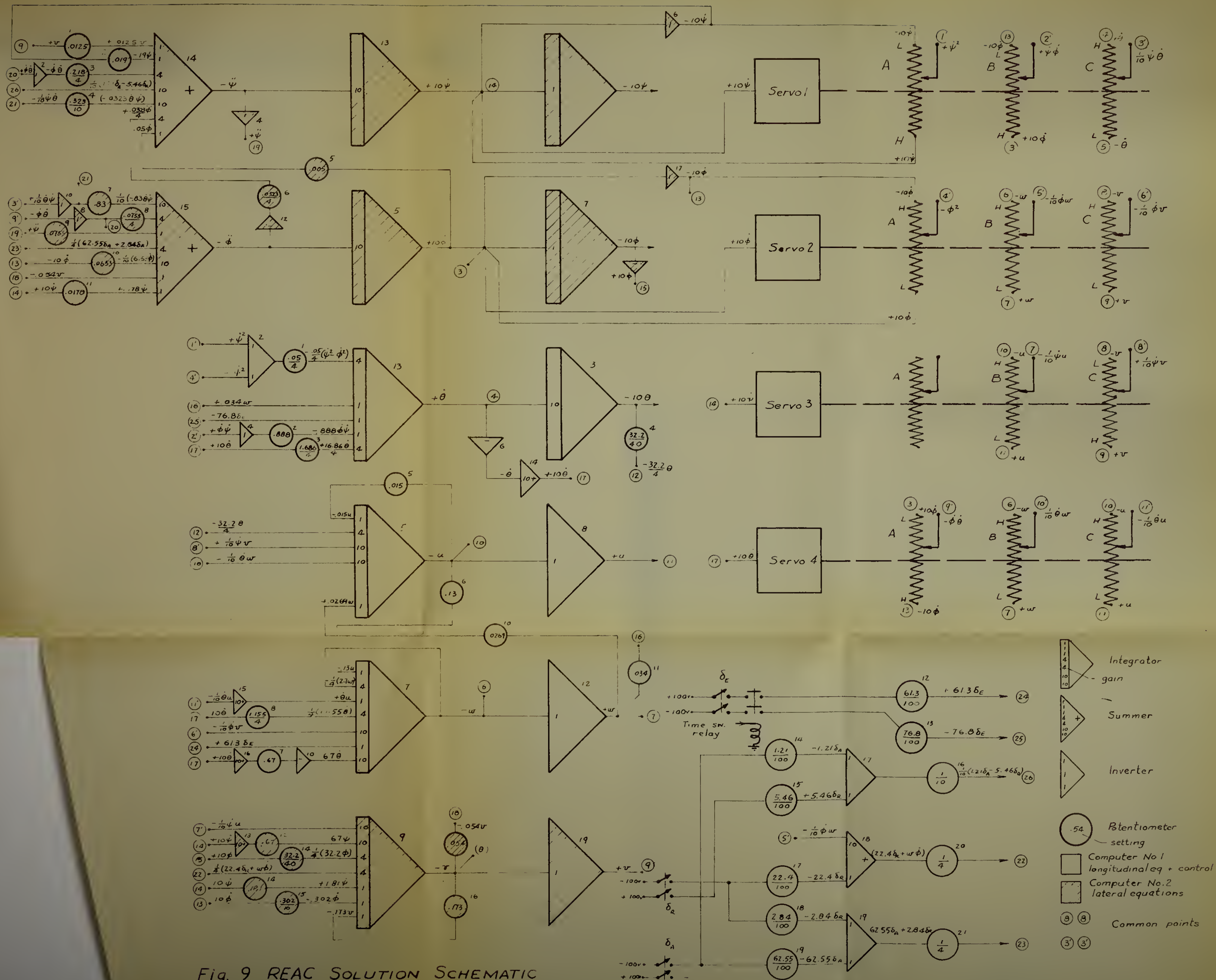
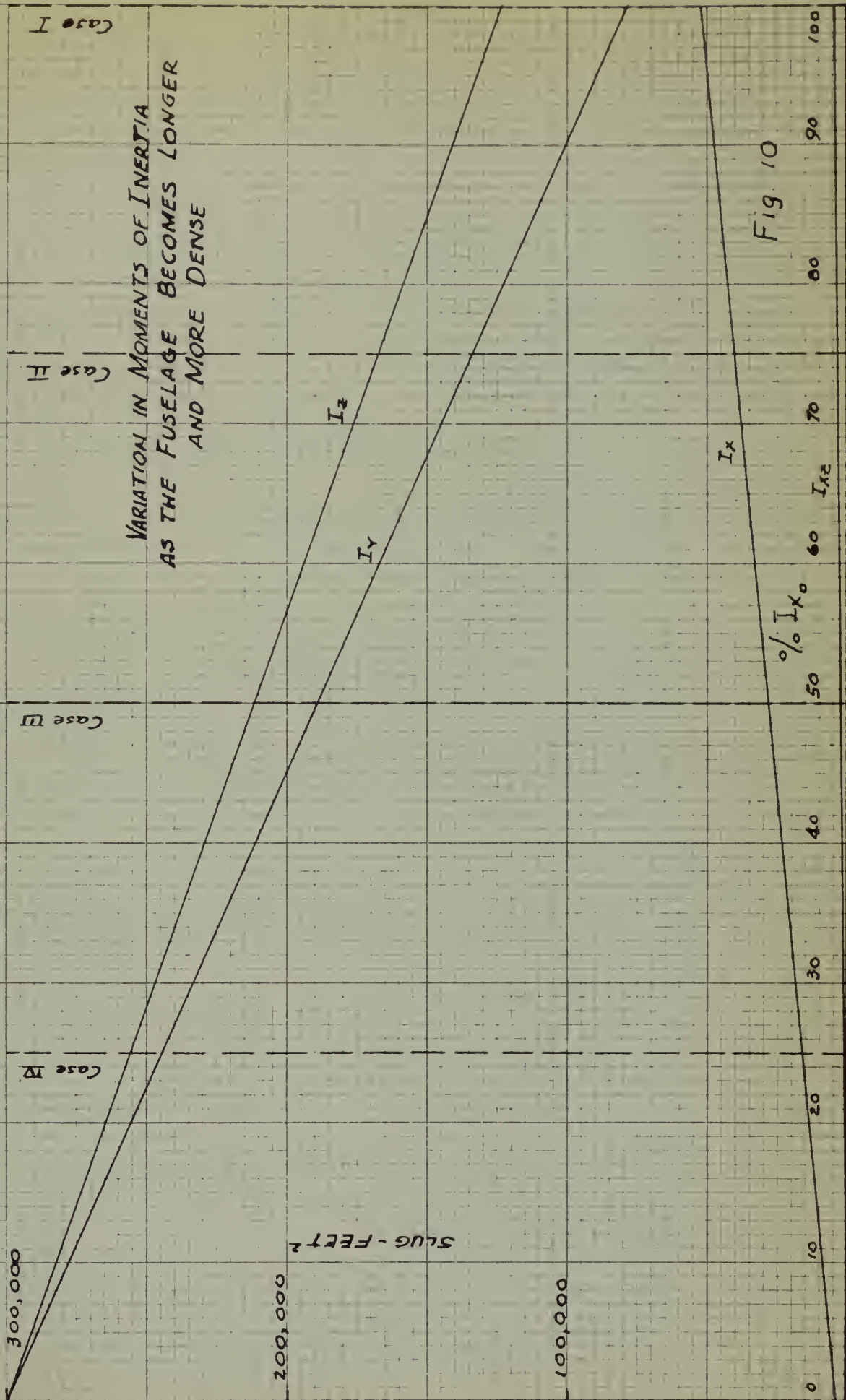


Fig. 9 REAC SOLUTION SCHEMATIC









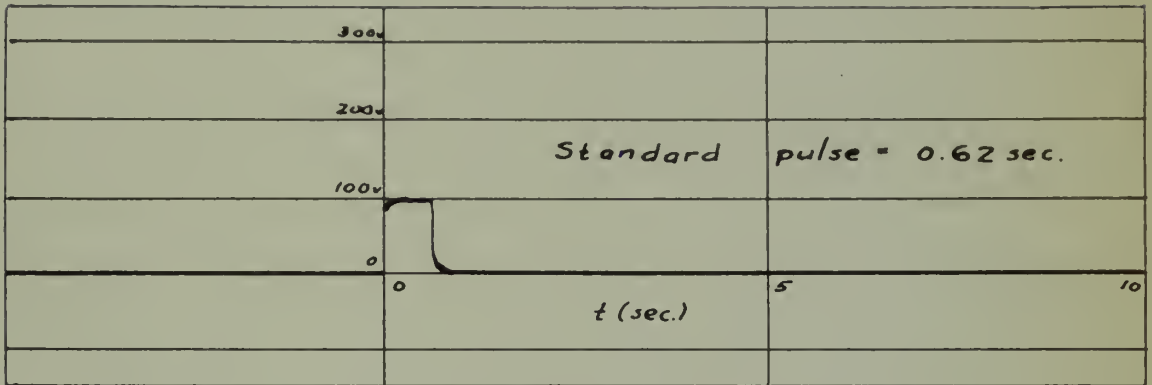


FIG. 11 Standard pulse function input.

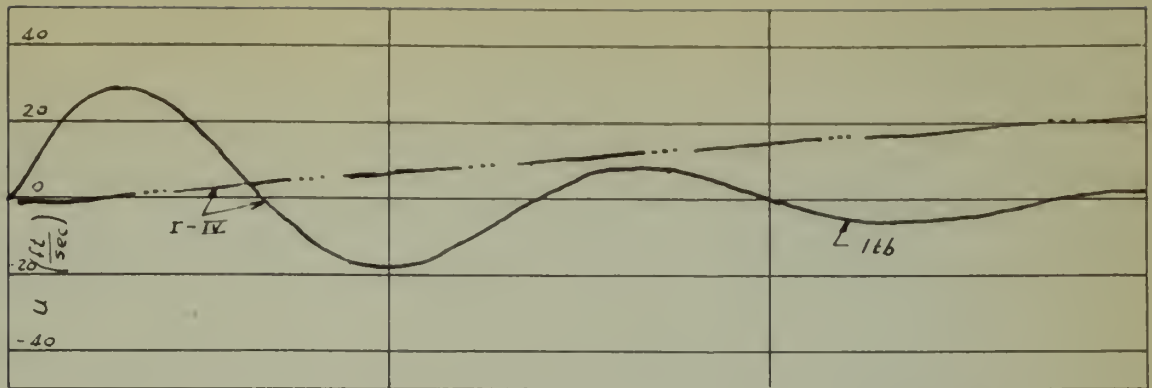
GENERAL INFORMATION ON THE REACTION OF THE REACTING HISTORY CURVES

THE POINT ABOVE WAS SUBMITTED TO THE RUDDER, ELEVATOR, OR AILERON SWITCH AT THE FORCING FUNCTION.

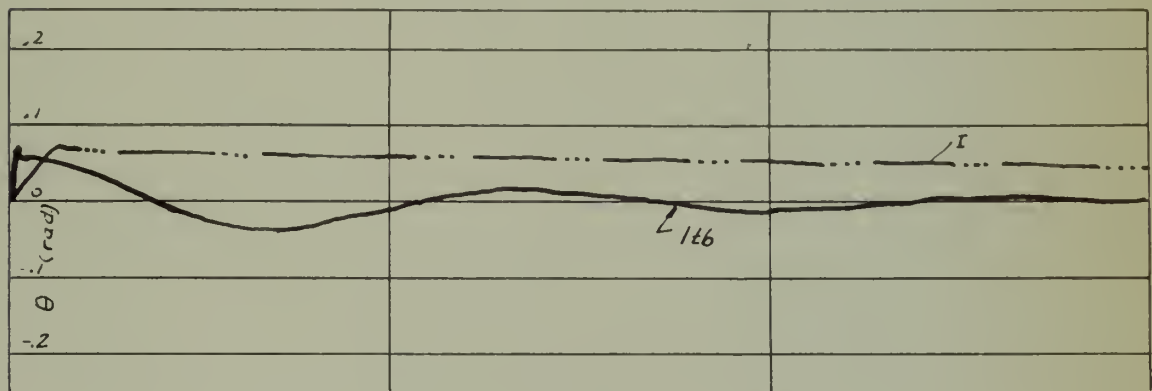
EIGHT CHANNELS WERE RECORDED IN THE HISTORY. IN THE PRESENT FIGURES THE A AND B PARTS OF THE FIGURES REPRESENT THE RIGHT CHANNELS ASSIGNED TO THE INDICATED FORCING FUNCTION.

THE FOUR RECORDED CURVES WERE NOT THE SAME LENGTH AS THEY WERE THE SAME LENGTH. THE CHANGING OF THE LENGTH OF THE CURVES FOR EACH CHANNEL IN THE VERTICAL DIRECTION. WHILE THE CURVES WERE NOT ALL THE SAME LENGTH THE CHANGING OF THE LENGTH OF THE CURVES WAS NOT THE SAME LENGTH AND THE CURVES WERE NOT THE SAME LENGTH.





$ltb = \text{long time base}$



0 5 10 15  
0 100 200 300

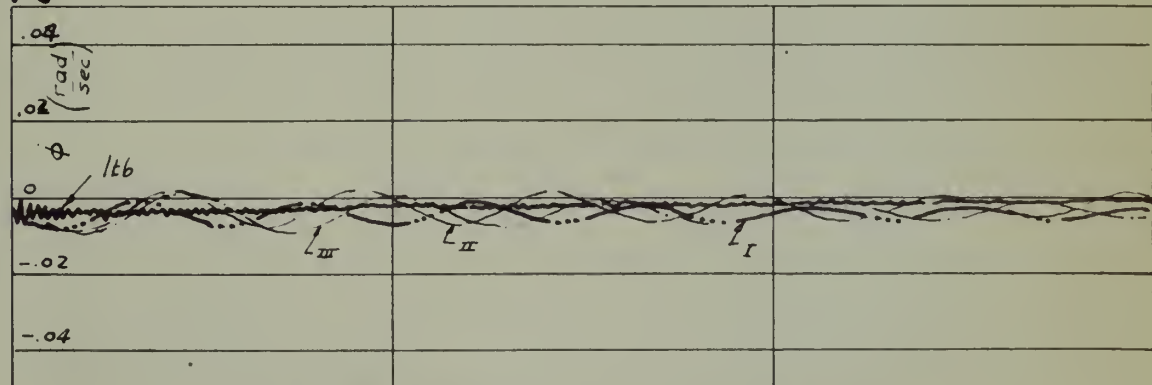
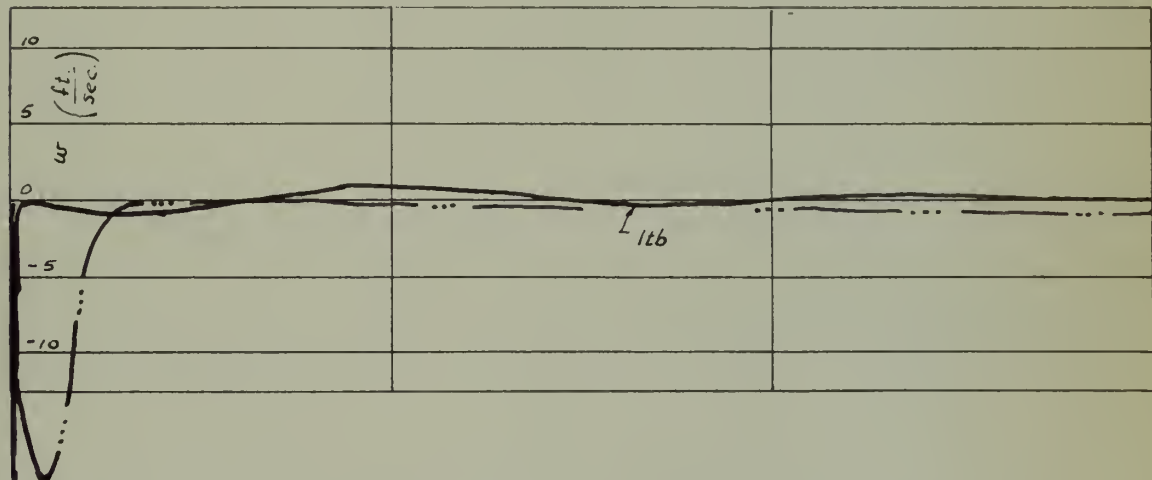
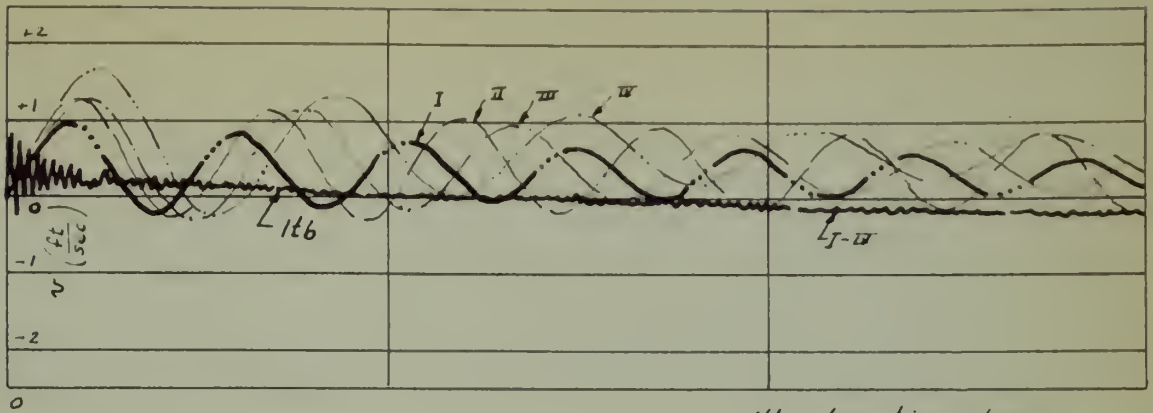


FIG. 124. RESPONSE OF AN ELEVATOR SYSTEM,  
CASE I, II, III AND IV (SEE 4.11).







ltb = long time base

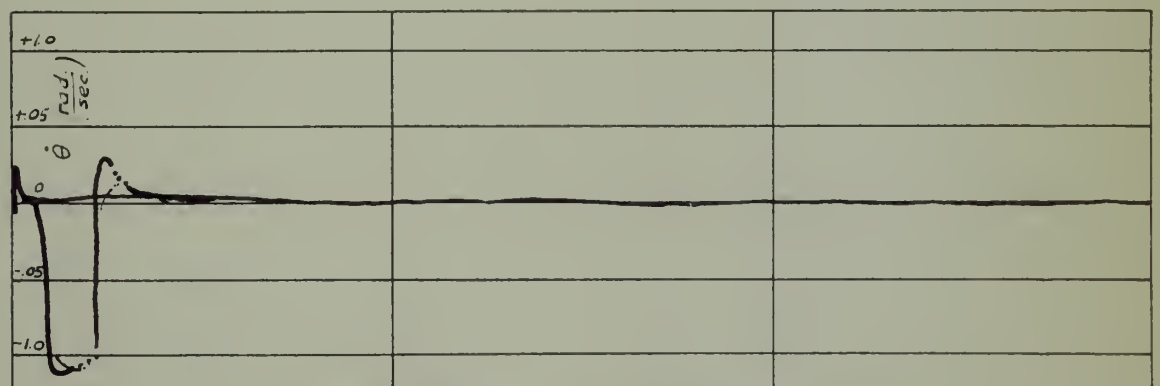
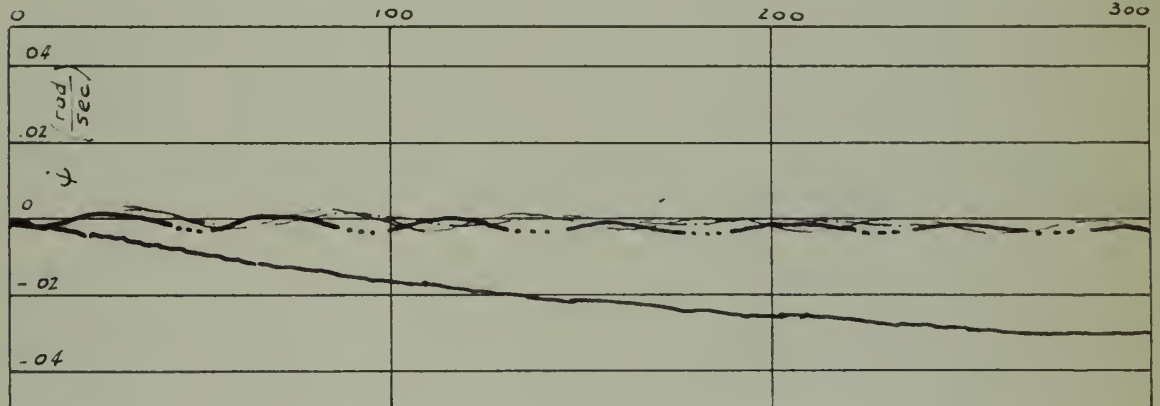
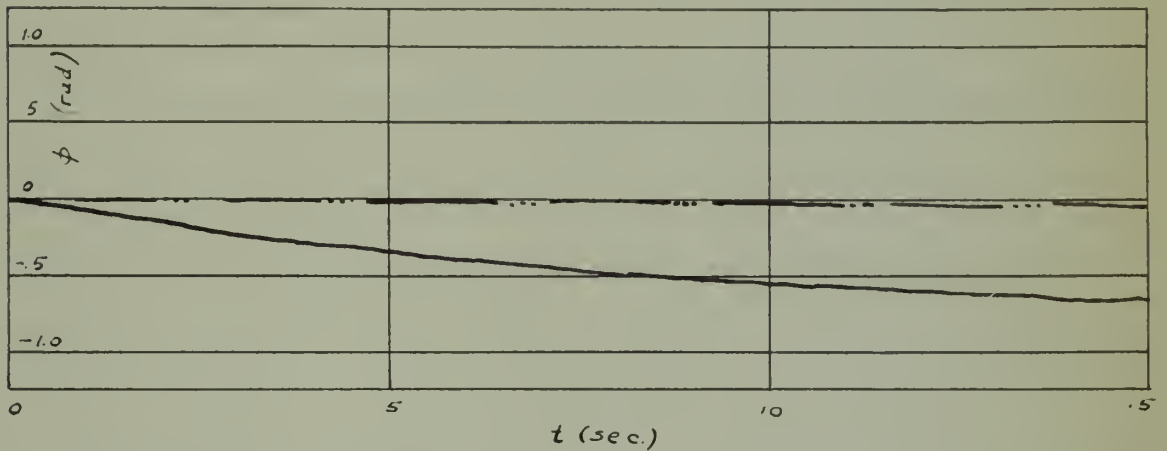


FIG. 1. 1—ROUTE TO AND FROM TARDIGRADES, CAGES I, II, III, AND IV (SEE LEVEL).



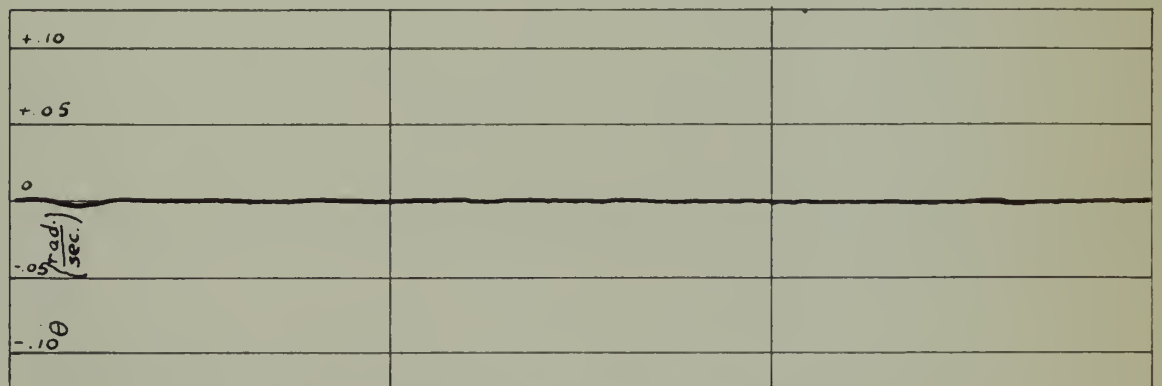
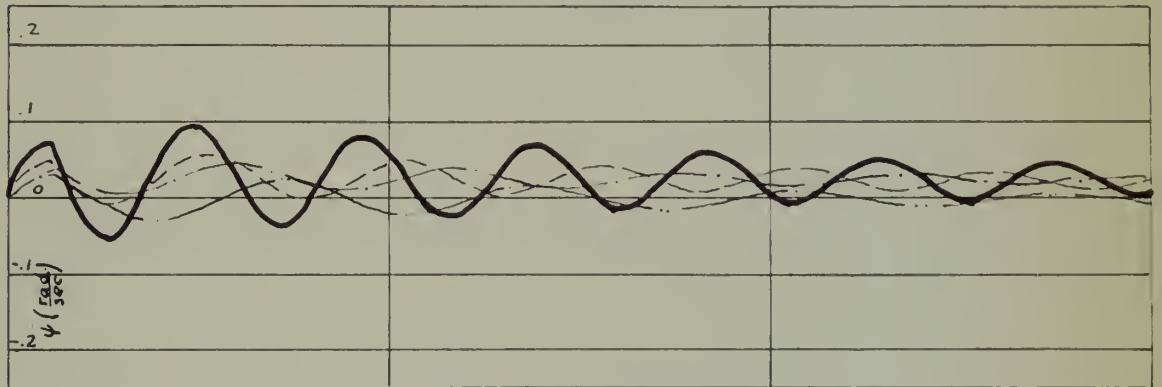
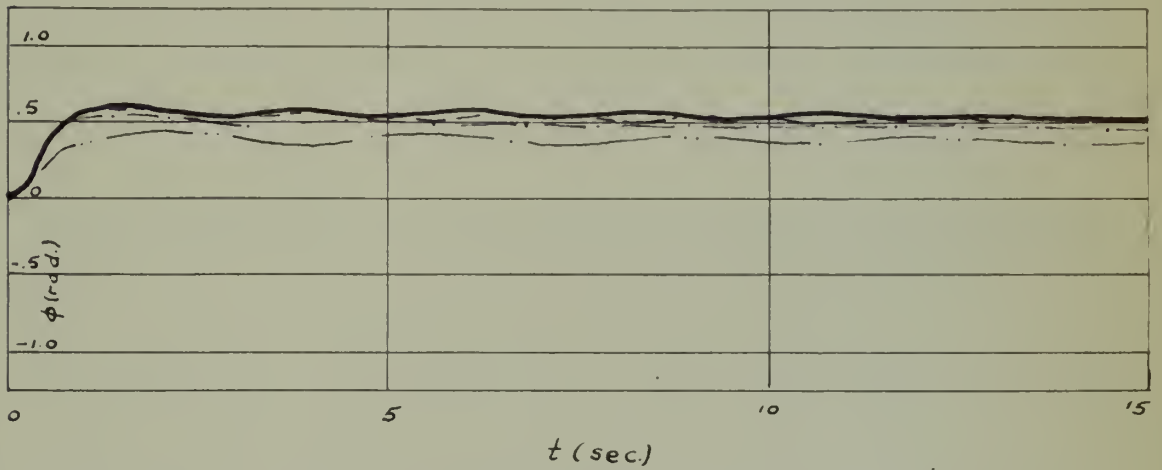
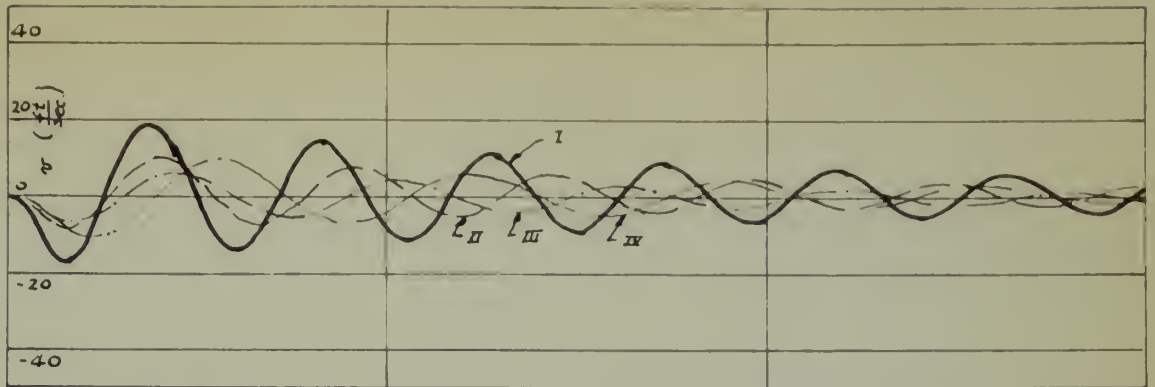


FIG. 1. TA-100000 TO AN -110000 P.W. S.,  
C-1000 I, II, III - 0.17 (5 A. 100 V L.).



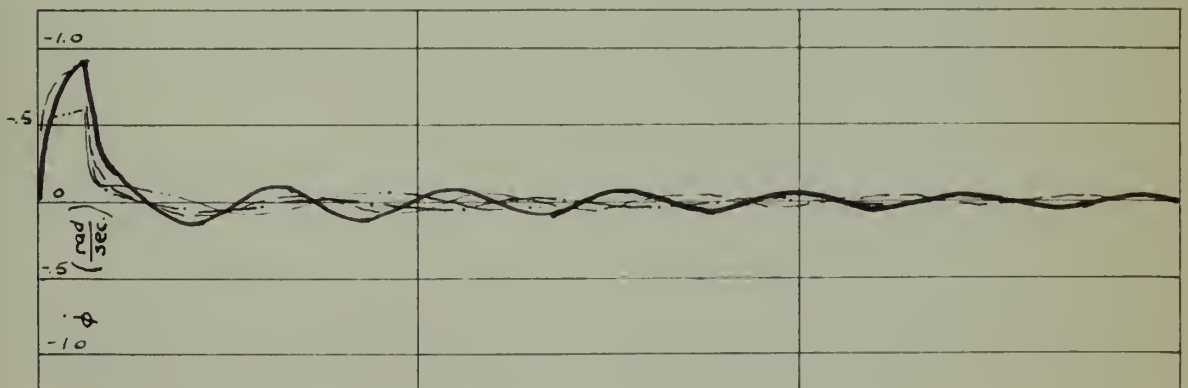
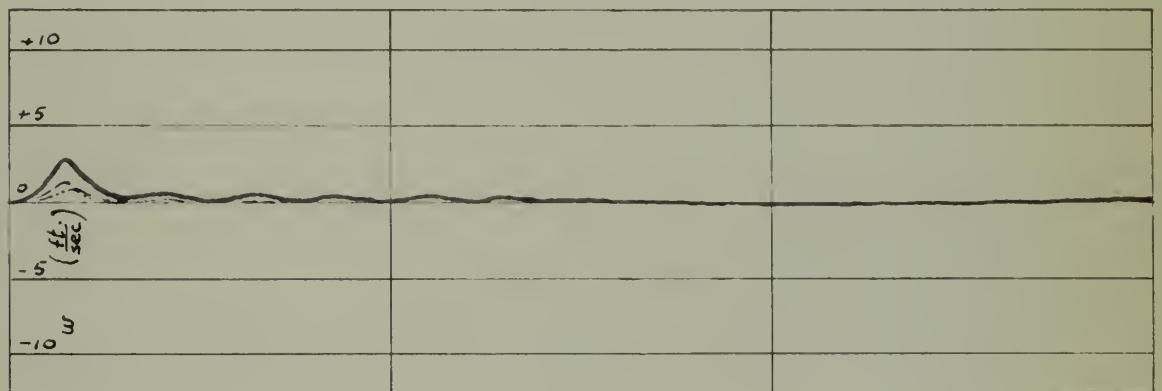
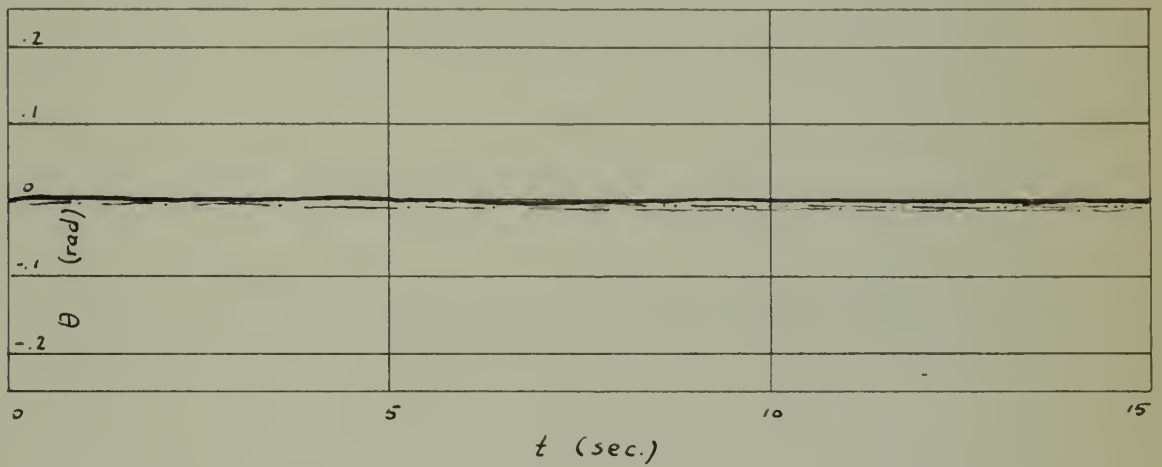
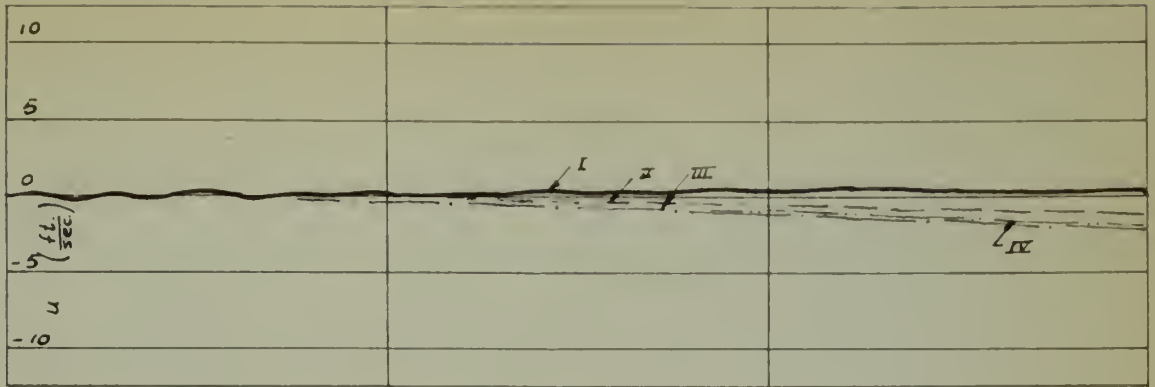


FIG. 123. RESPONSE TO A STEADY MOTION, FOR CURVES I, II, III, AND IV (SEE PAGE 54).





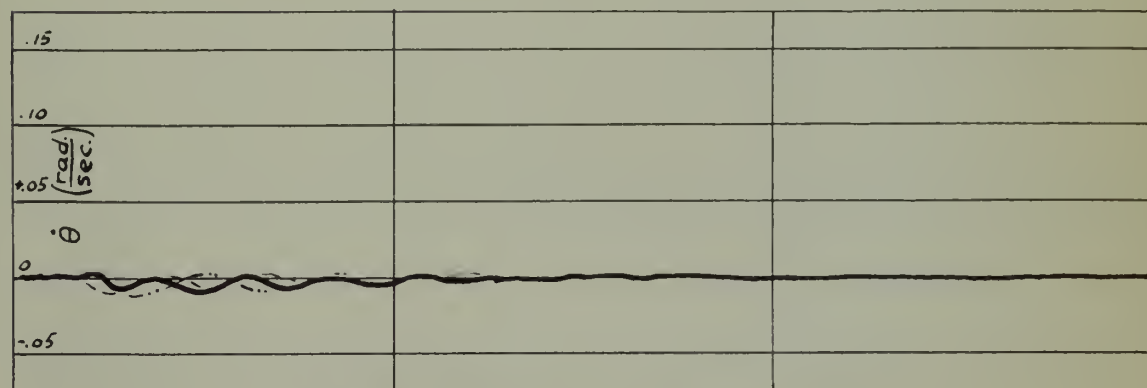
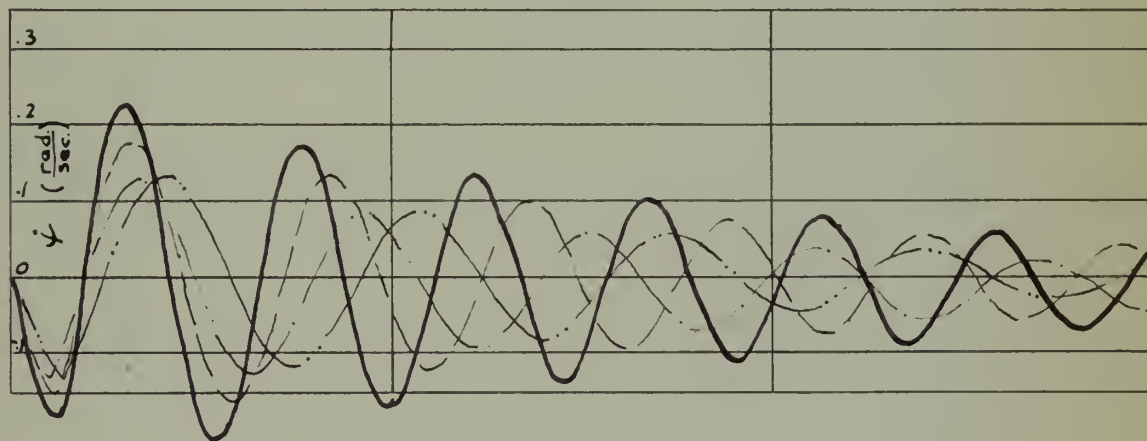
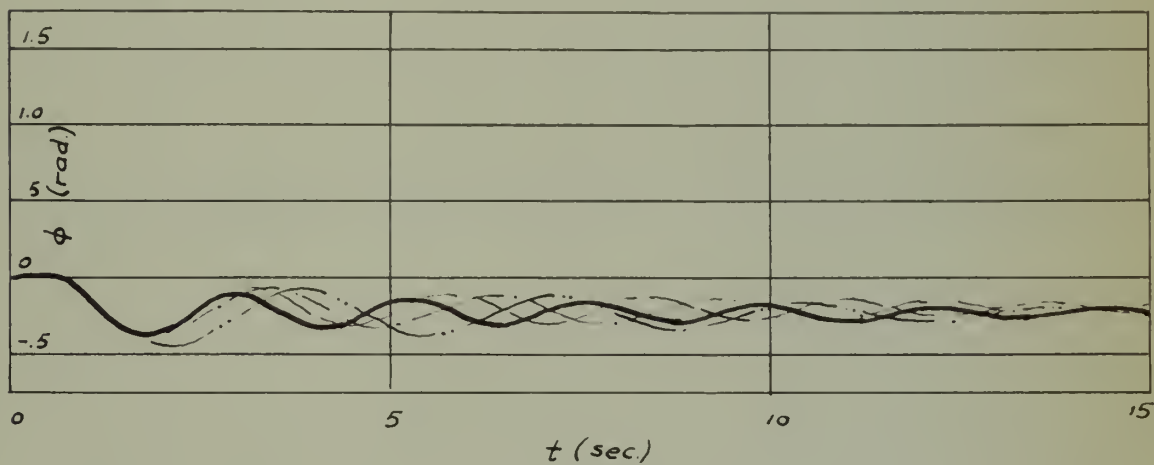
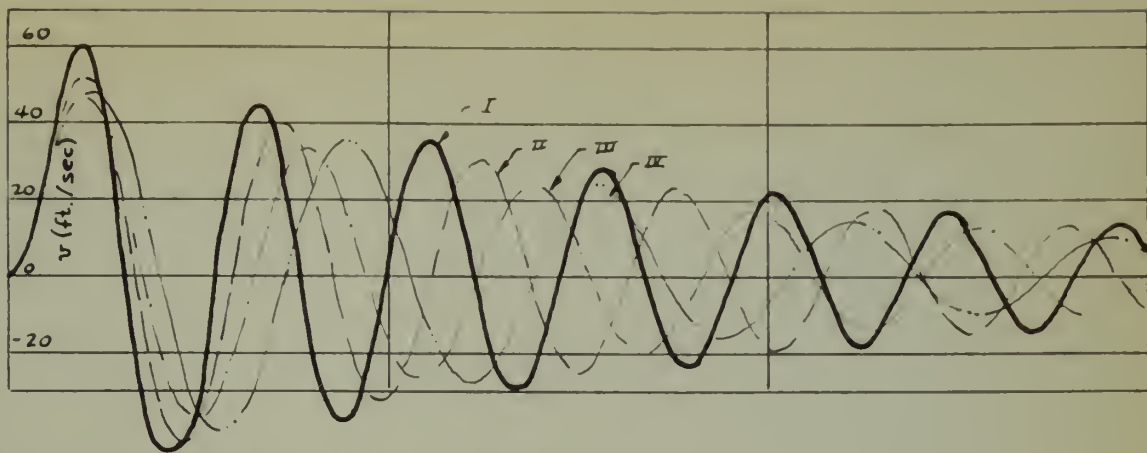


FIG. 14. RESPONSE OF A ROTARY SYSTEM,  
Cases I, II, III, AND IV (See A-11-10).



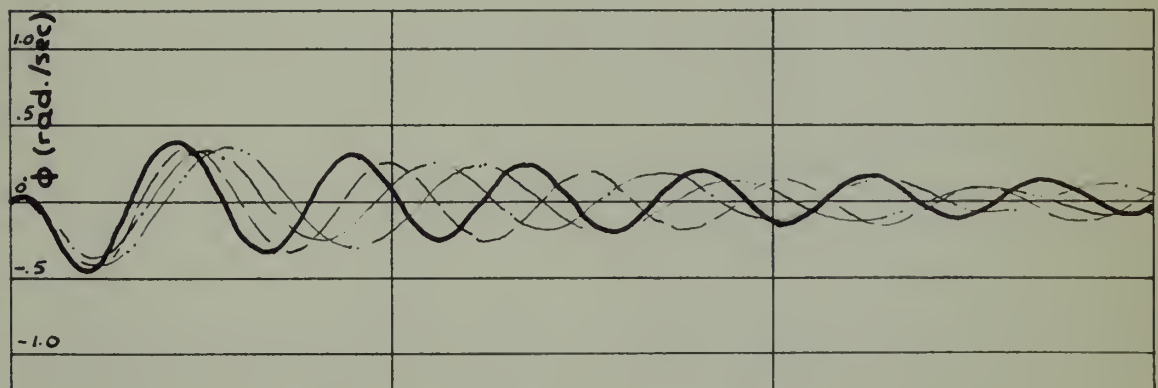
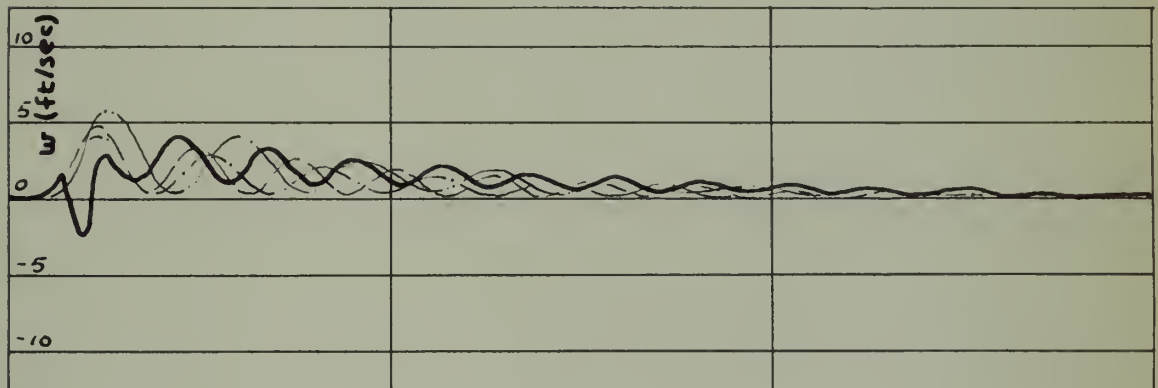
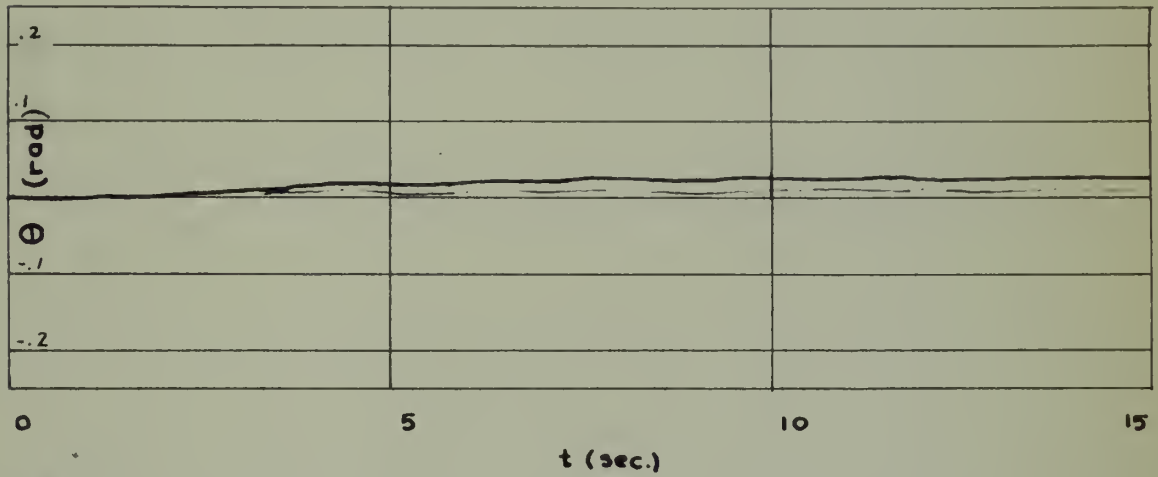
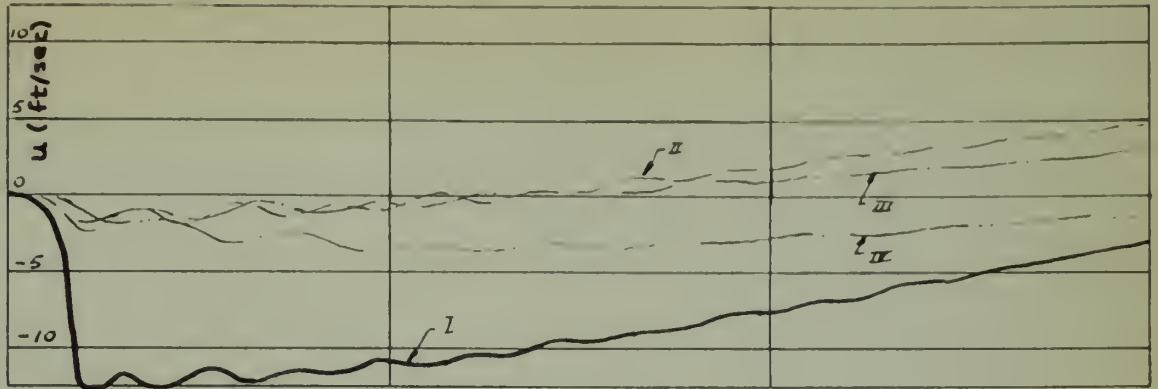
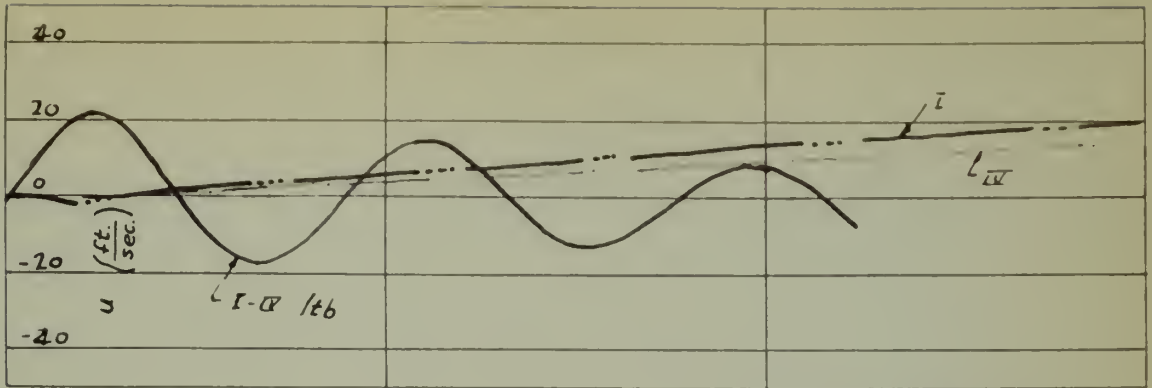


FIG. 14b. RESPONSE TO A ROCKET PULSE,  
CASES I, II, III, AND IV (SEA LEVEL).





$t_b = \text{long time base}$

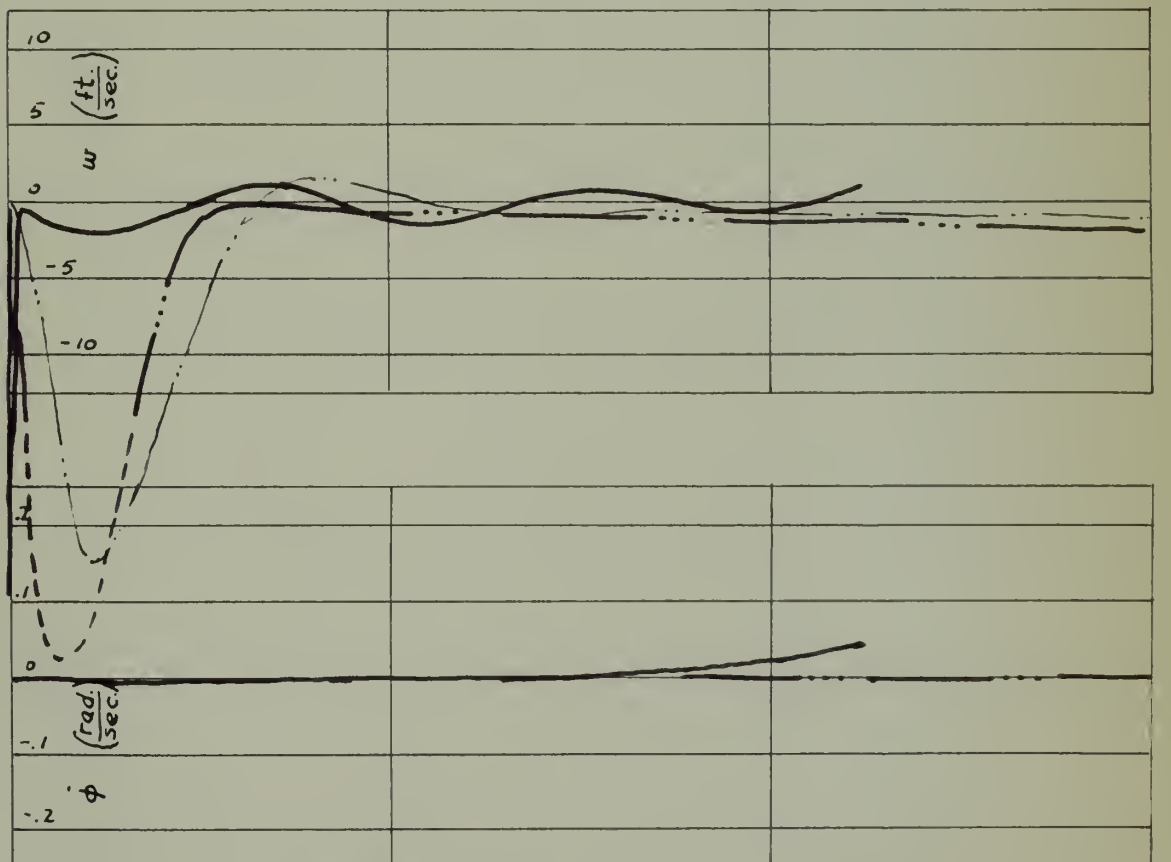
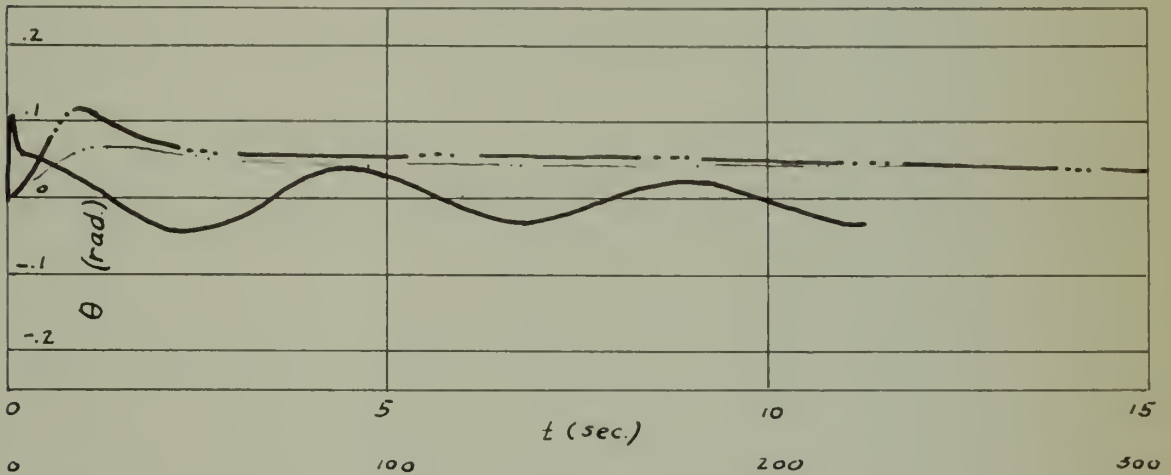
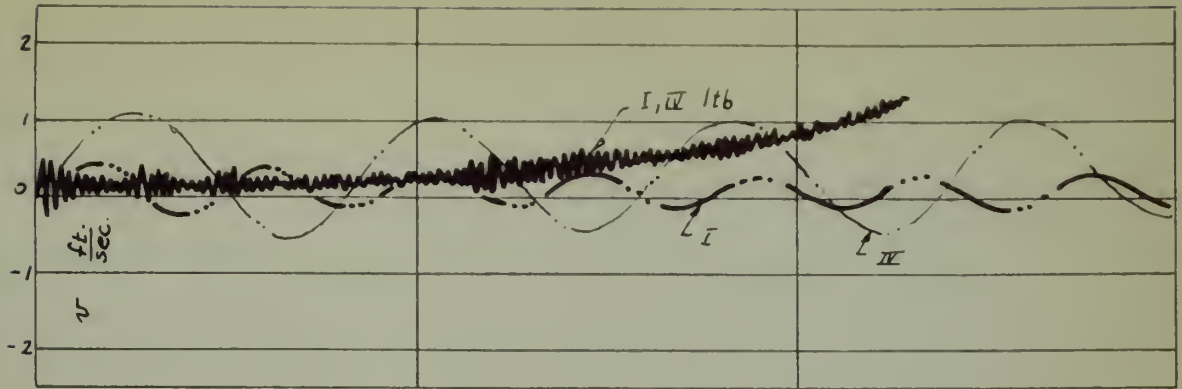


FIG. 10. VIBRATION OF A VERTICAL PILE,  $2000 \times 10^6$  LBS. (1:1000 FT. L.)





ltb = long time base

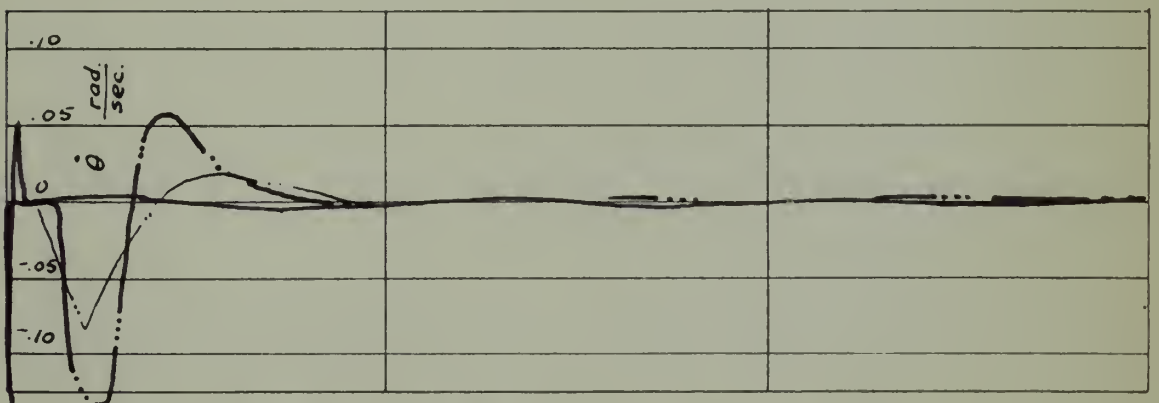
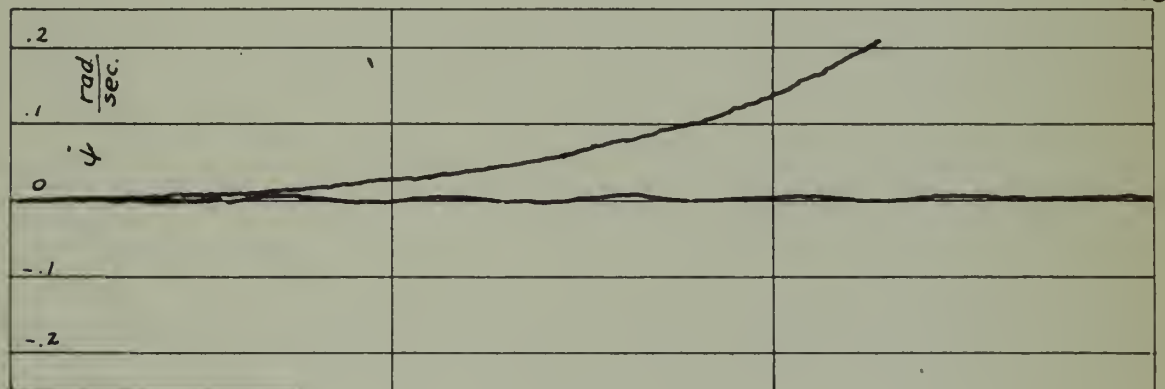
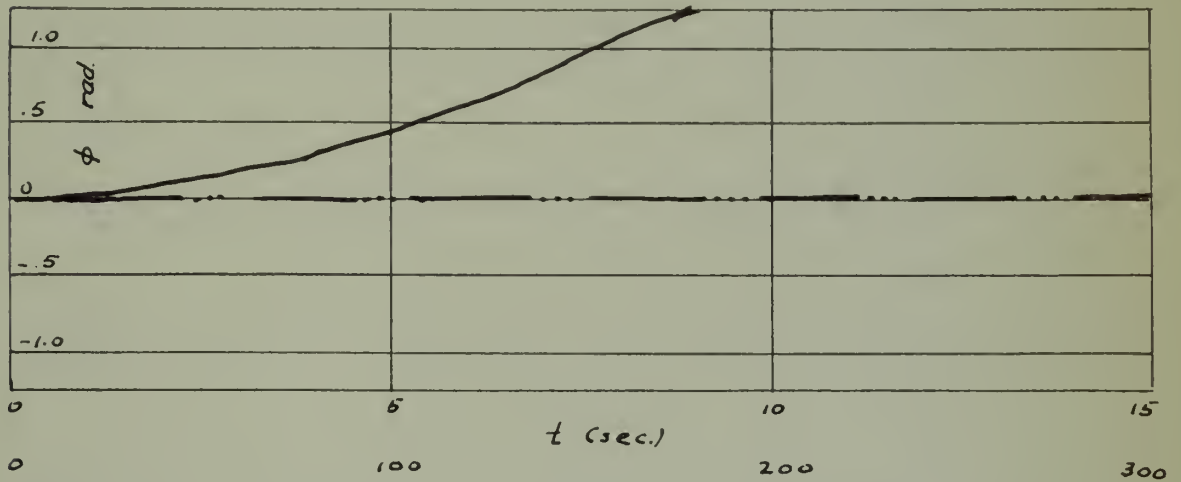


FIG. 10. RESPONSE TO A UNIT STEP, 2ND-ORDER MODIFIED (4, 100-T.).





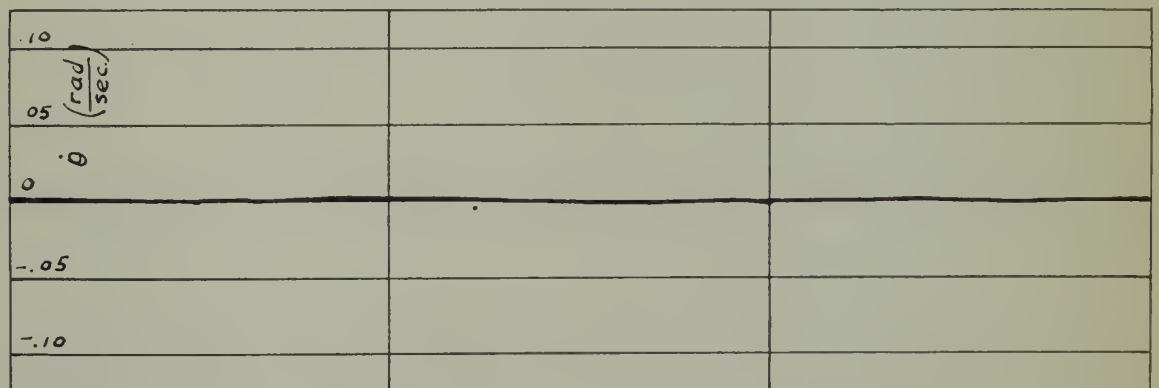
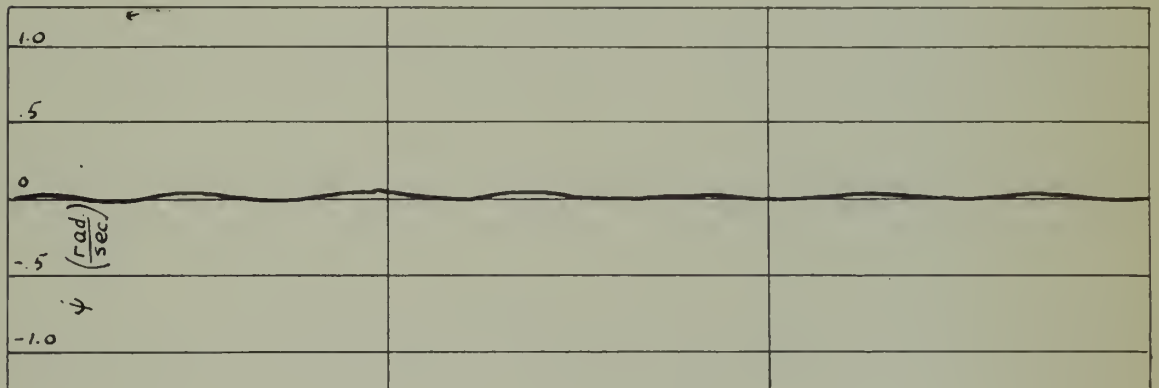
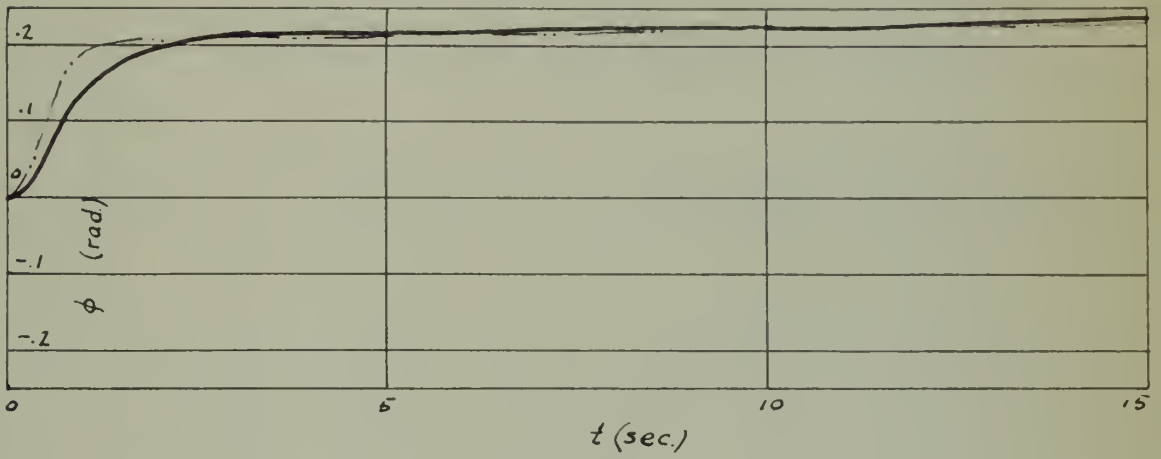
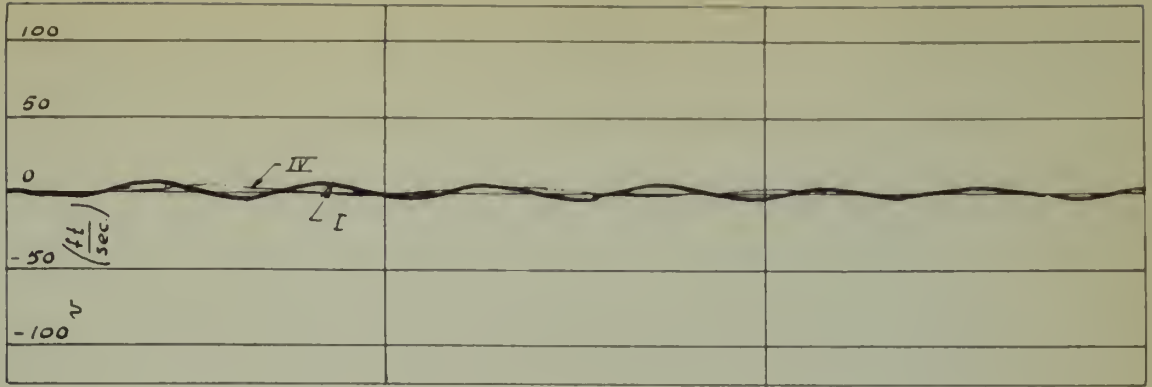


FIG. 1. The response of the system to a step input of 0.1 rad/sec, with a damping ratio of 0.05 (1.5%).



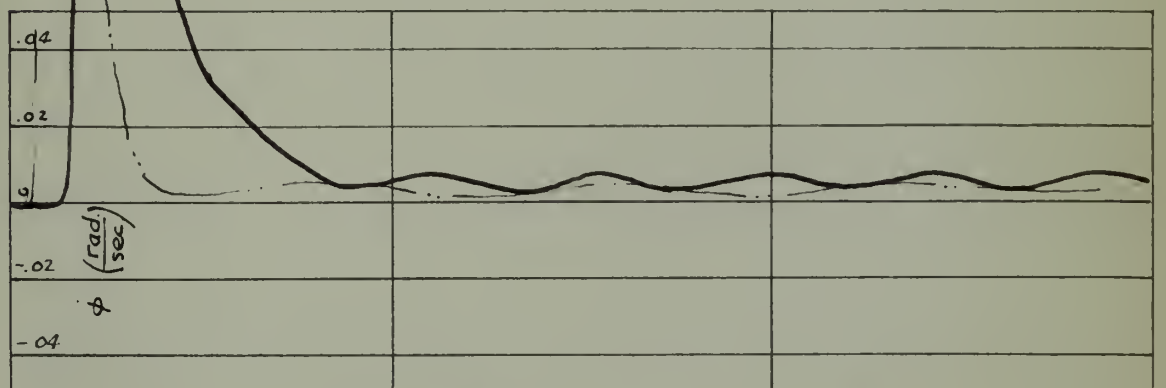
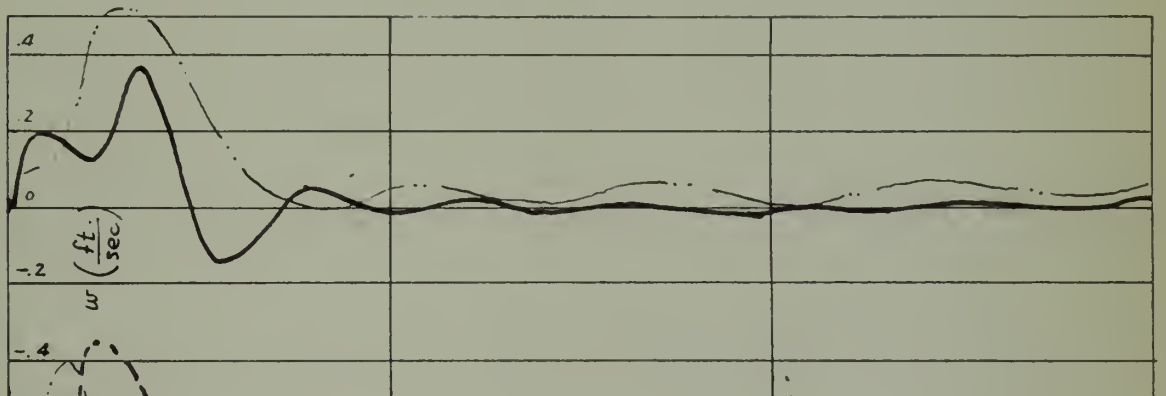
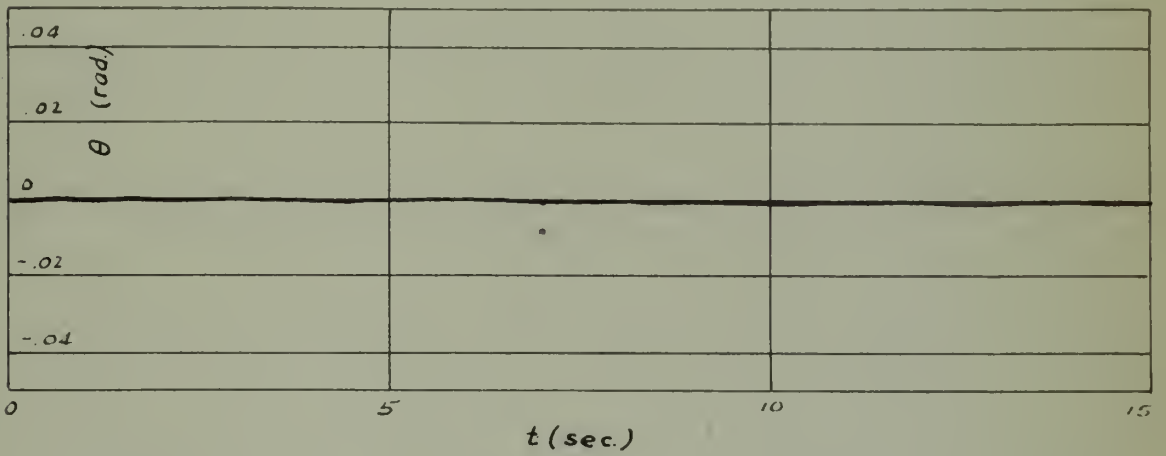
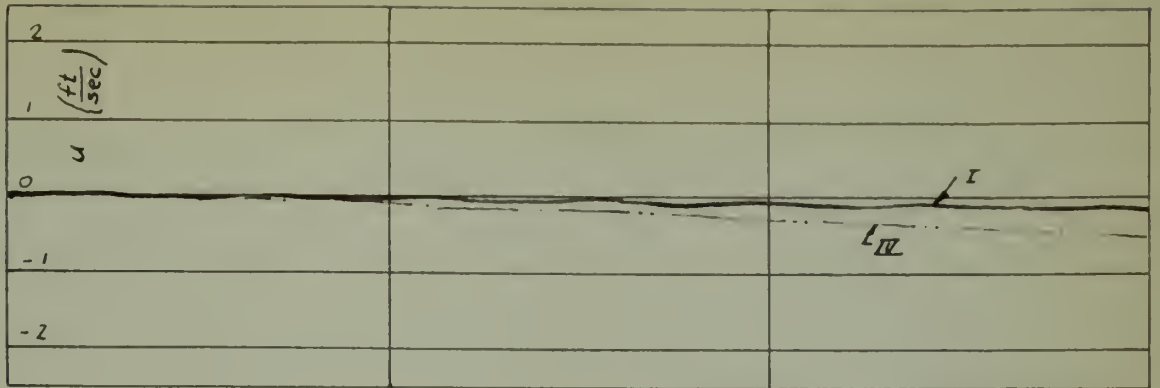


FIG. 108 RESPONSE TO AN IMPULSE FORCE.  
DATA I AND IV CORRESPOND TO 1000 CY.



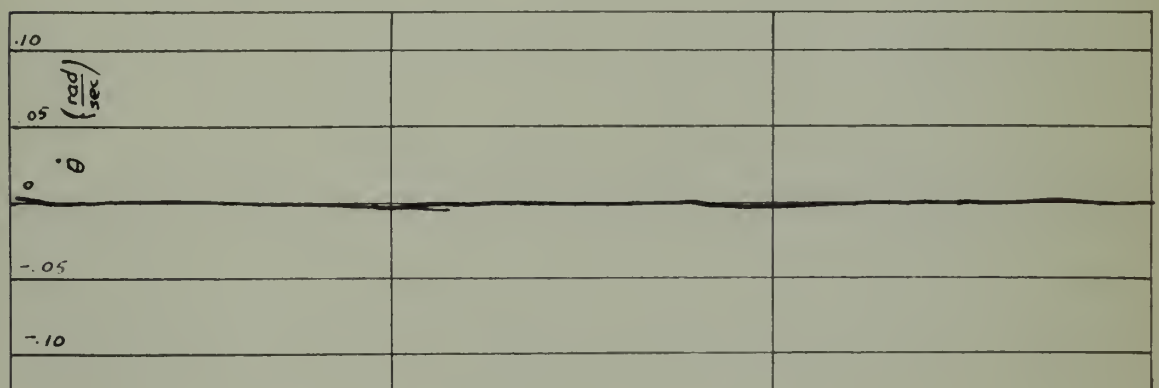
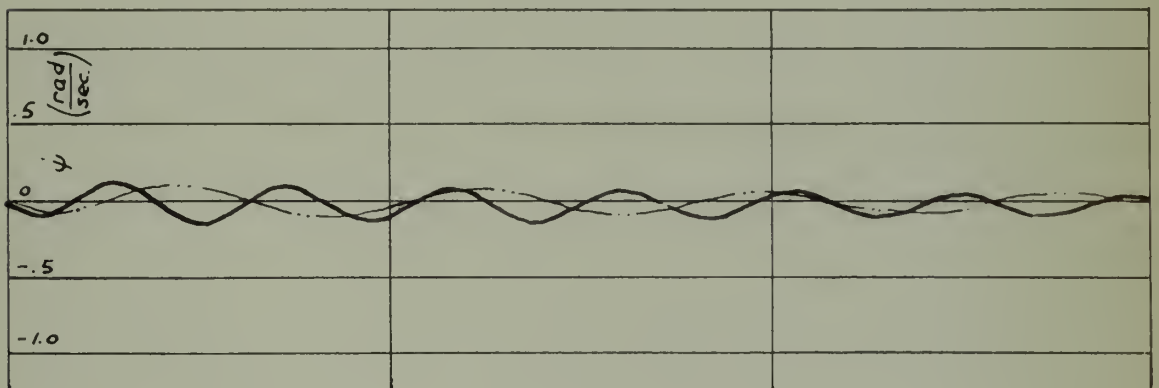
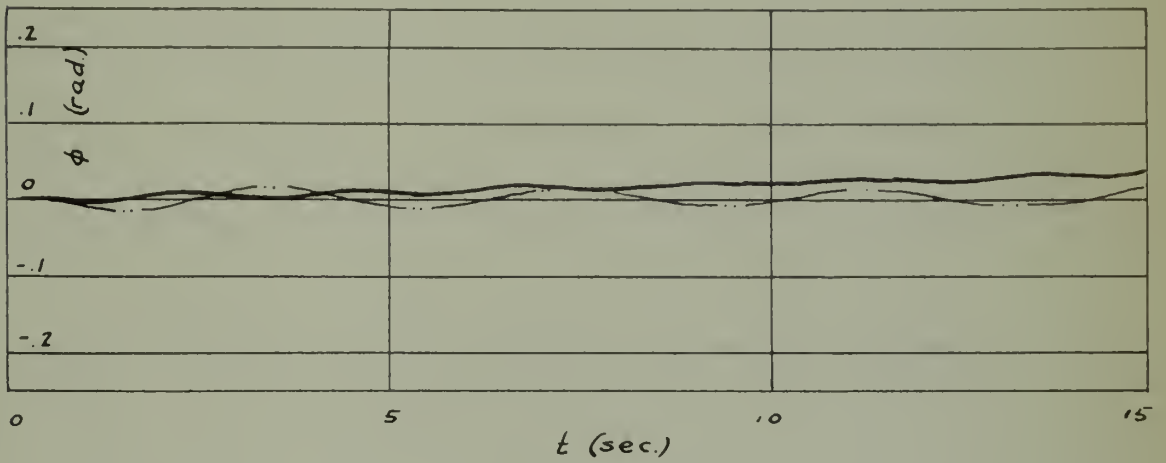
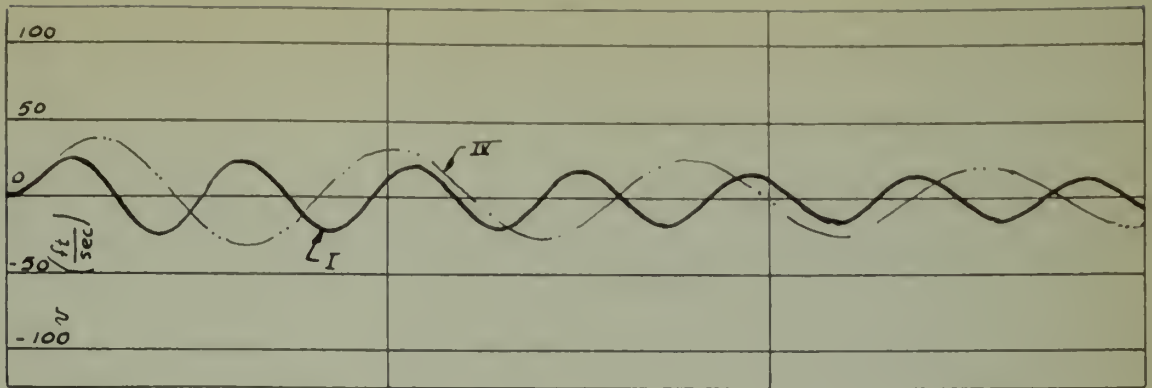


FIG. 17A. RESPONSE TO A RESTING POINT.  
CASES I AND II. INITIALS (45, 0.00) (1.0, 0.0).





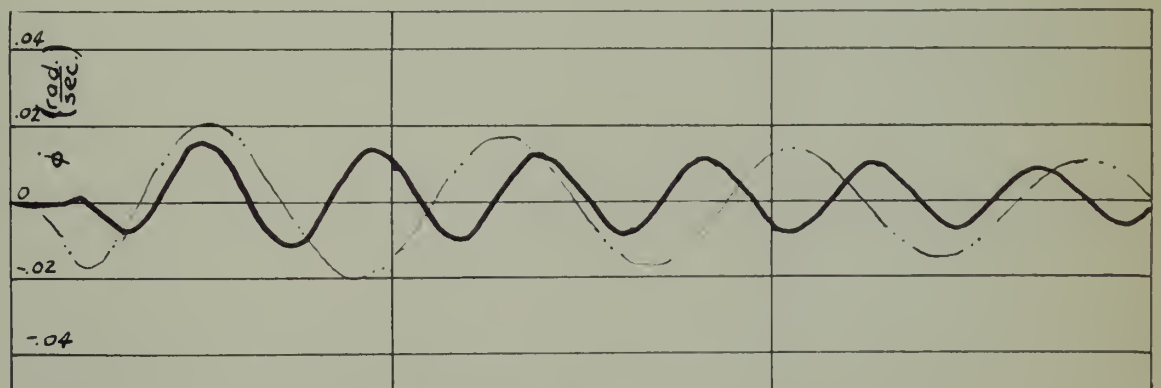
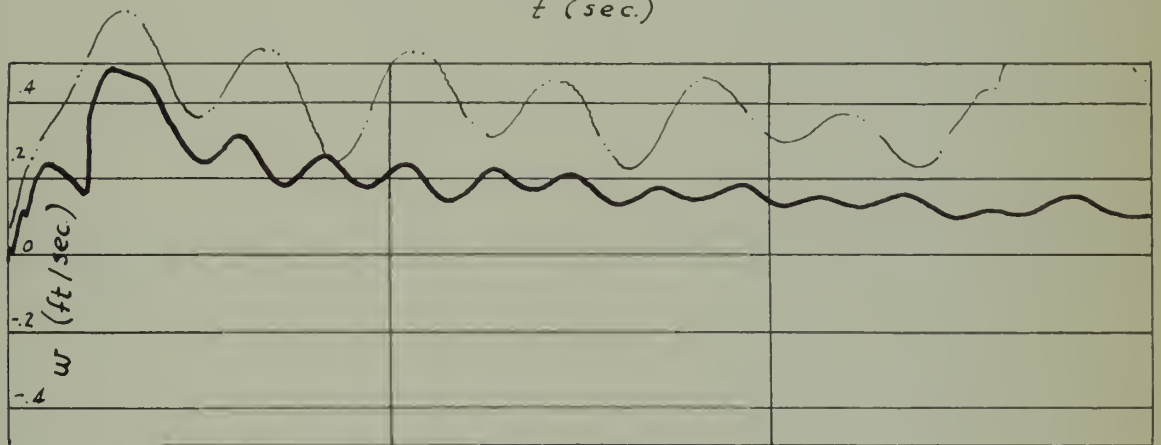
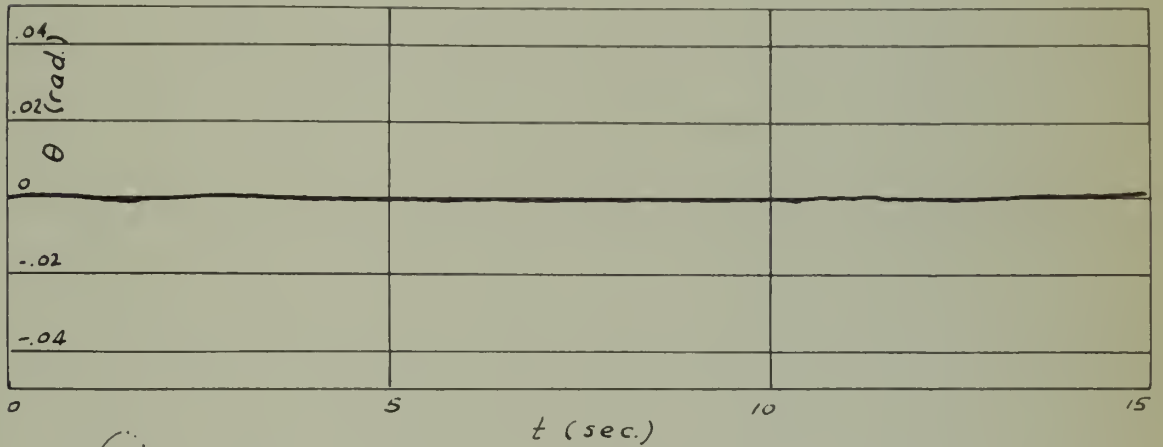
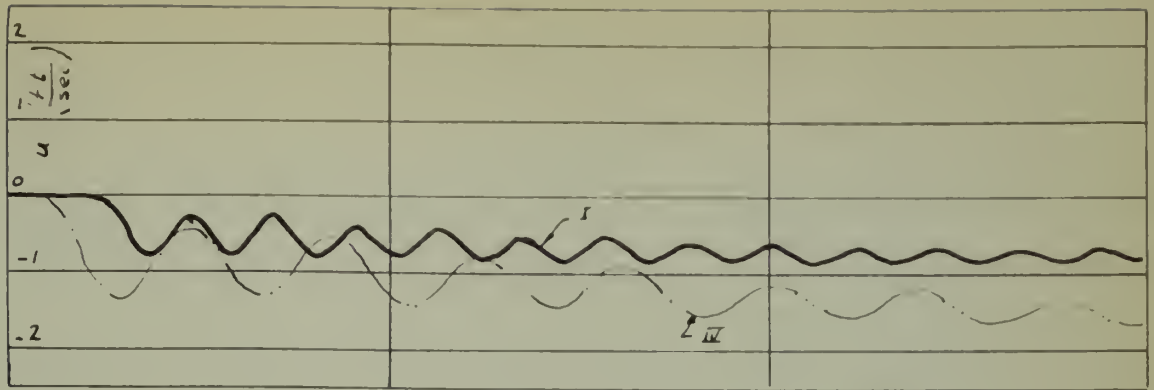
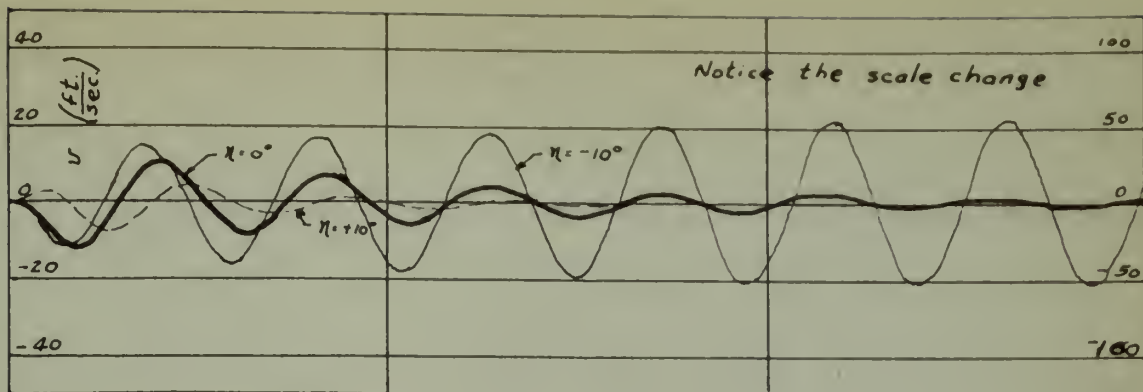


FIG. 1. Response of a single-degree-of-freedom system to a sinusoidal input,  $\omega = 1.0$  rad/sec,  $\phi = 0.1$  rad.





$\eta = 0^\circ$

$\eta = \pm 10^\circ$

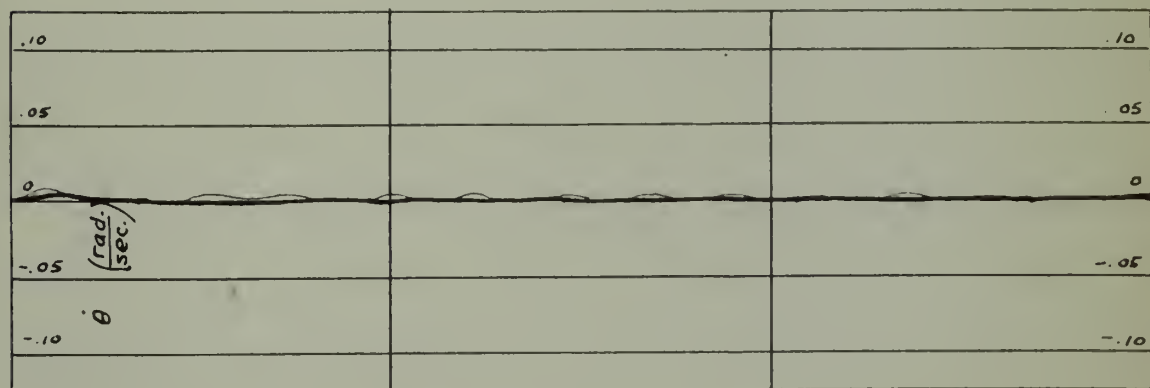
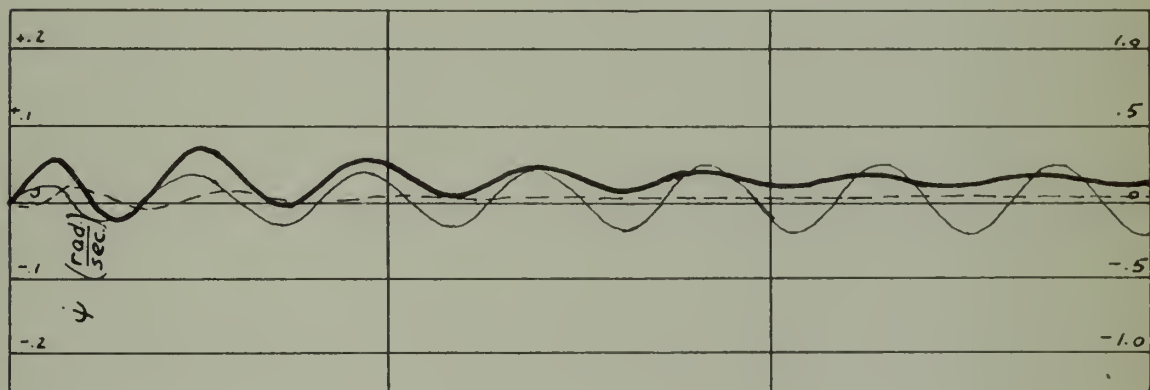
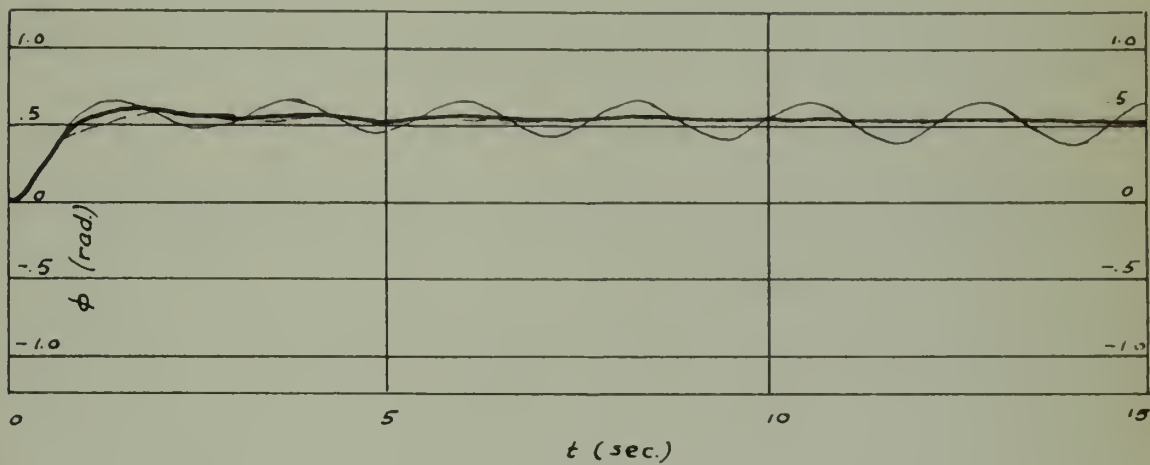
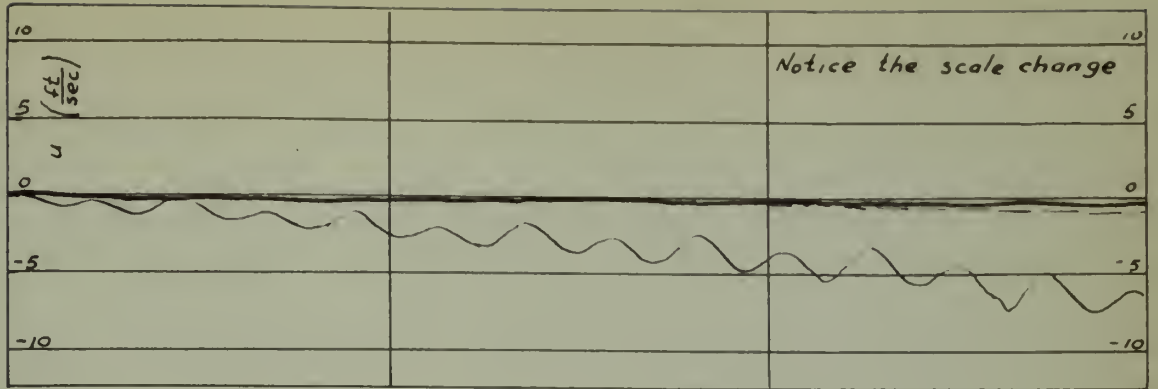


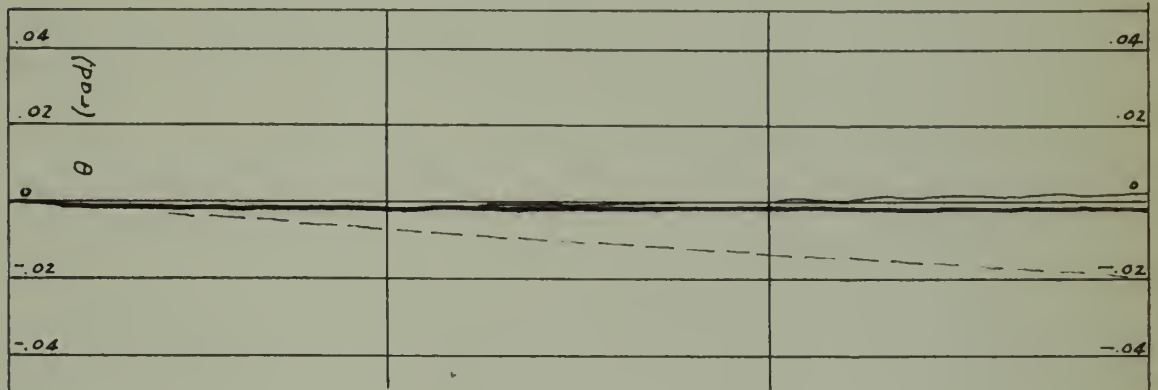
FIG. 1. A. RESPONSE TO A HARMONIC TONE.  
 $\eta = 0^\circ, \eta = 10^\circ, \eta = -10^\circ$  (5% LEVEL).





$\eta = 0^\circ$

$\eta = \pm 10^\circ$



0 5 10 15  $t$  (sec.)

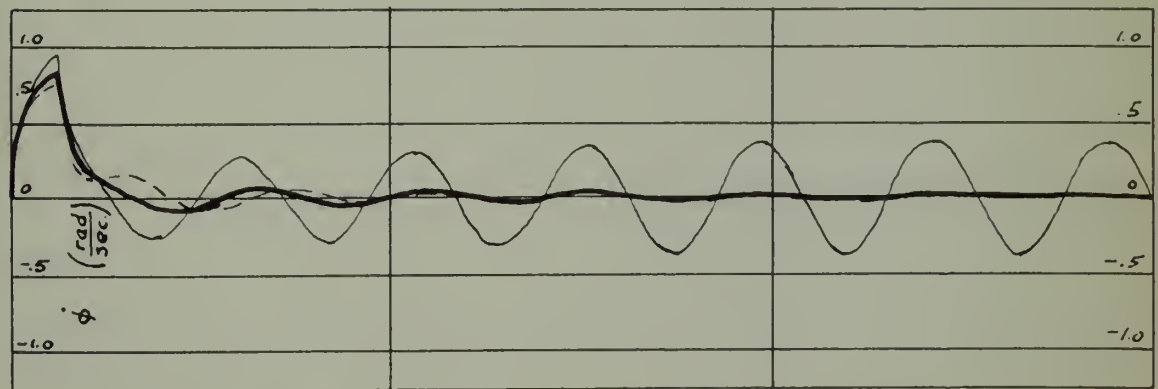
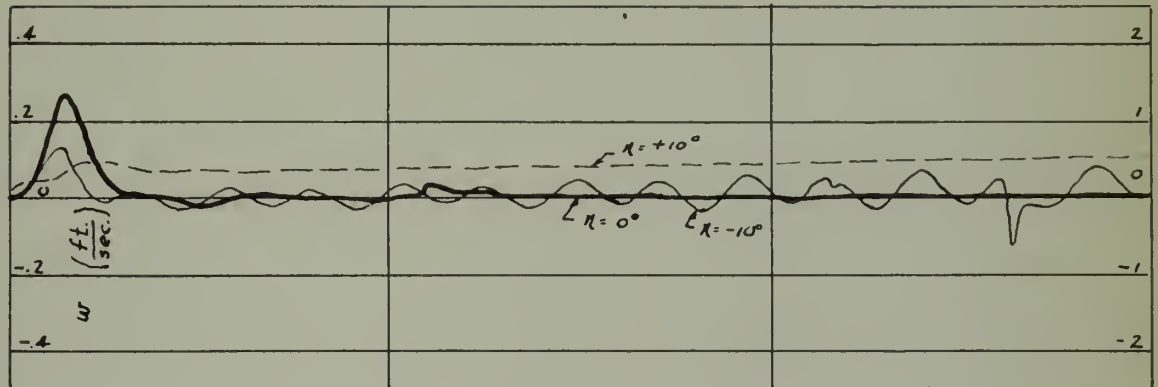
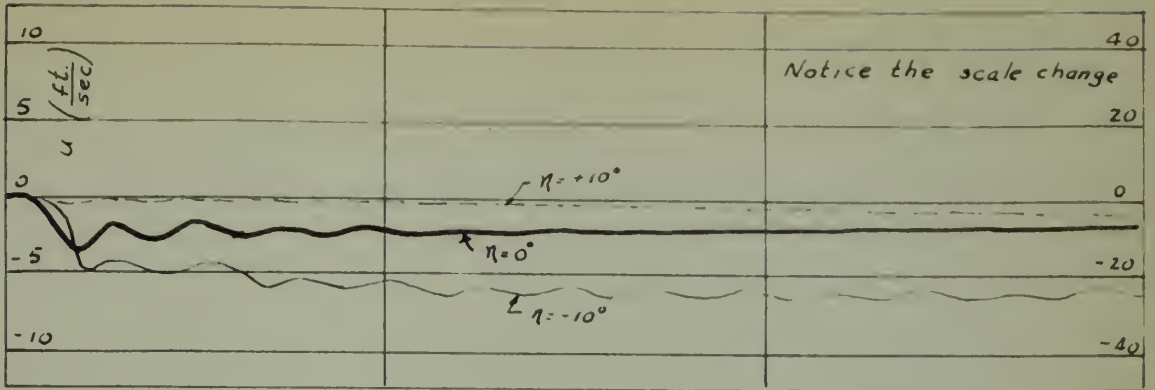


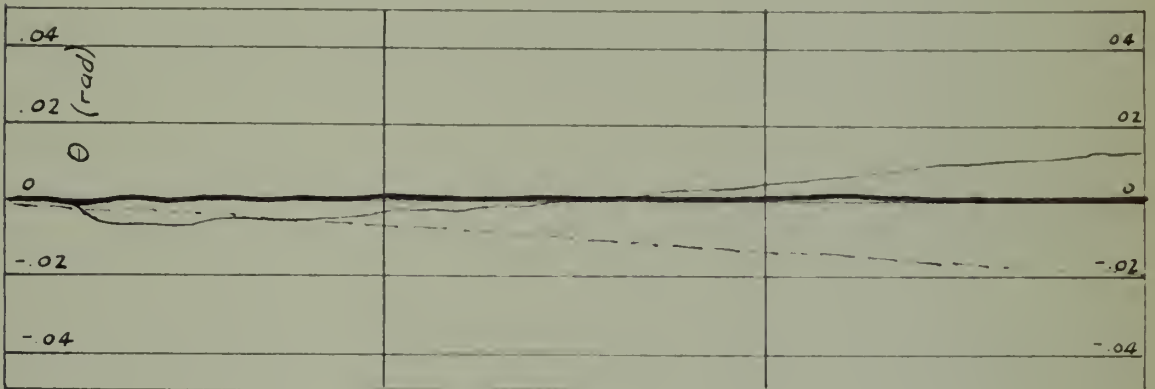
FIG. 115. FREE-OSCILLATION RESPONSE,  $\eta = 0^\circ, \eta = +10^\circ, \eta = -10^\circ$  (SEE FIG. 114).





$\eta = 0^\circ$

$\eta = \pm 10^\circ$



0 5 10 15  $t$  (sec)

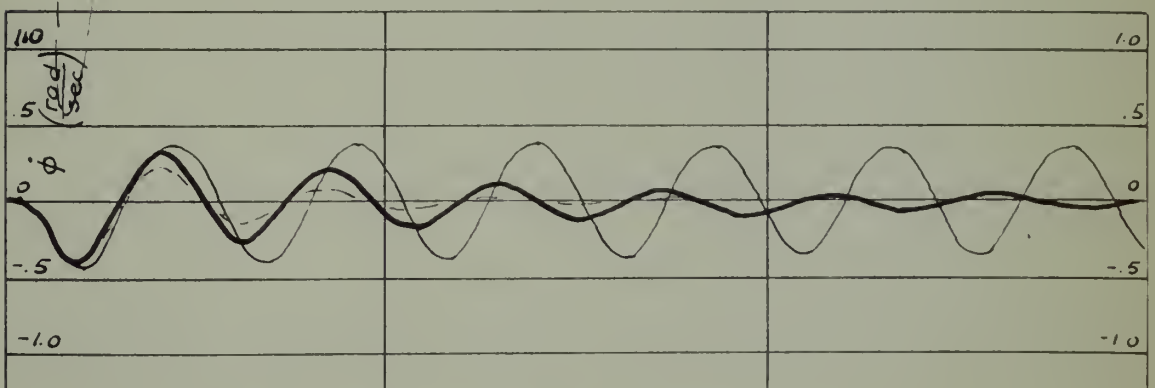
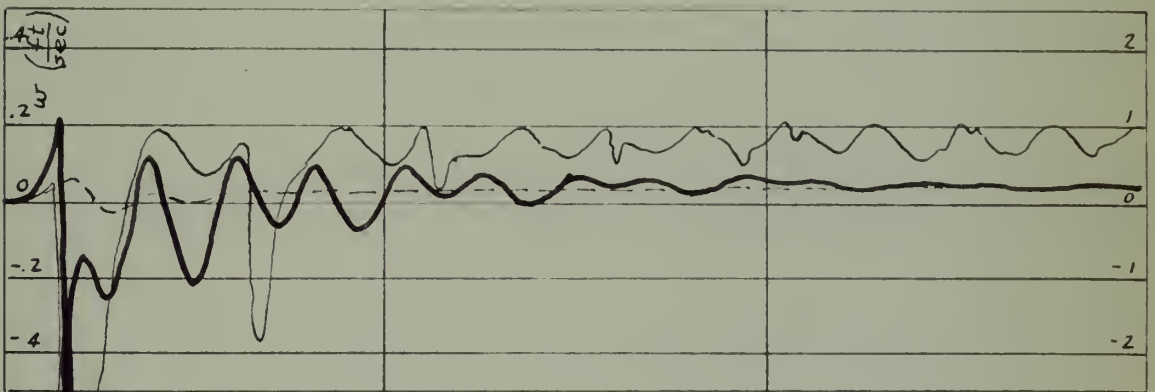
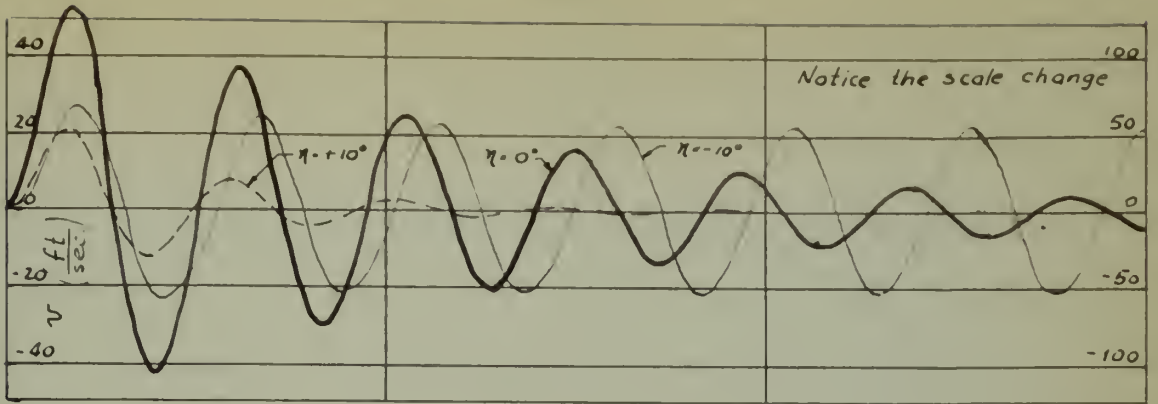


FIG. 1A.  $\eta = +10^\circ, 0^\circ, -10^\circ$  (GLA 40 V. 1).







$\eta = 0^\circ$

$\eta = \pm 10^\circ$

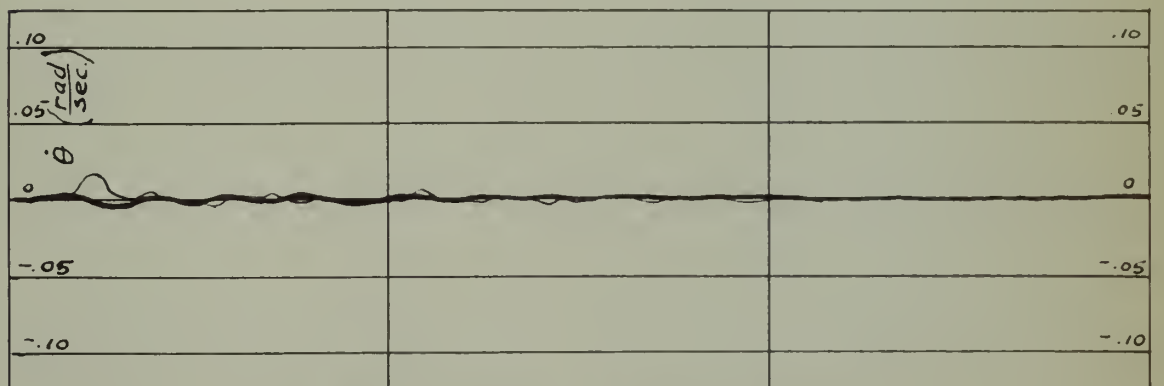
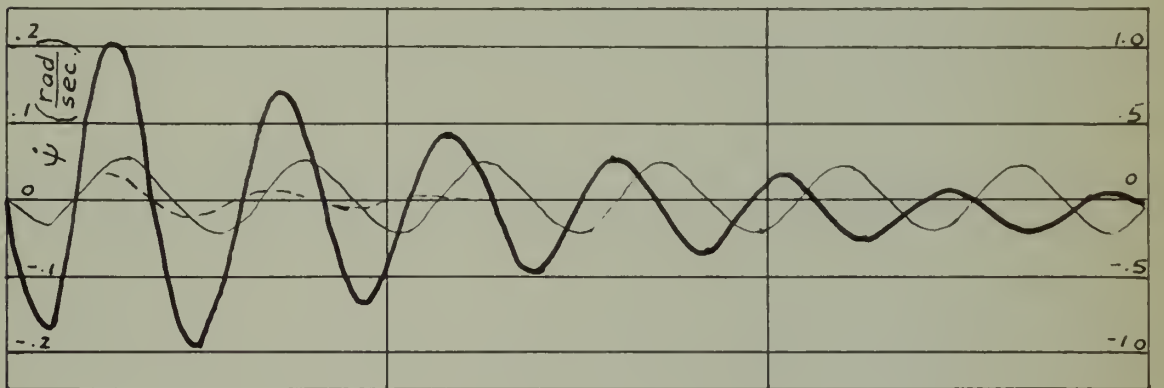
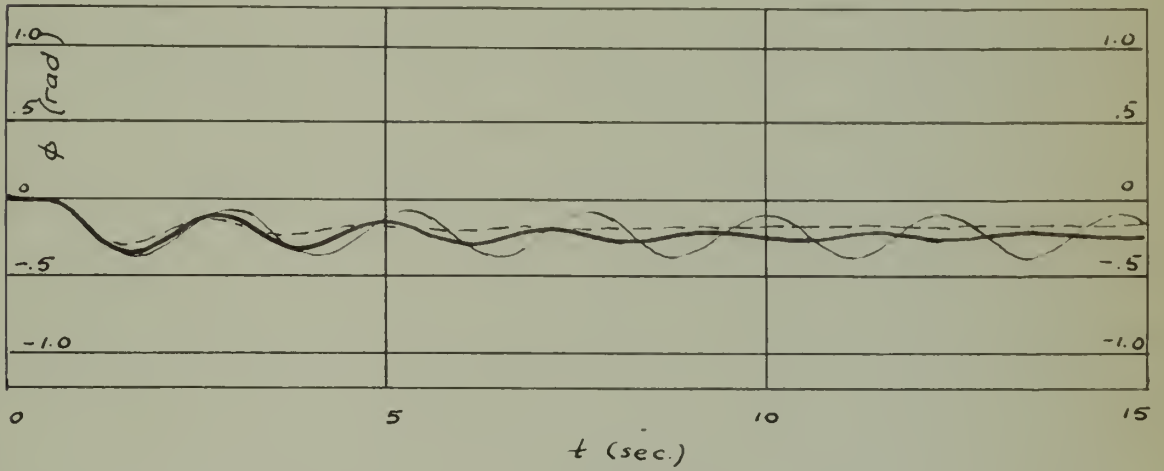
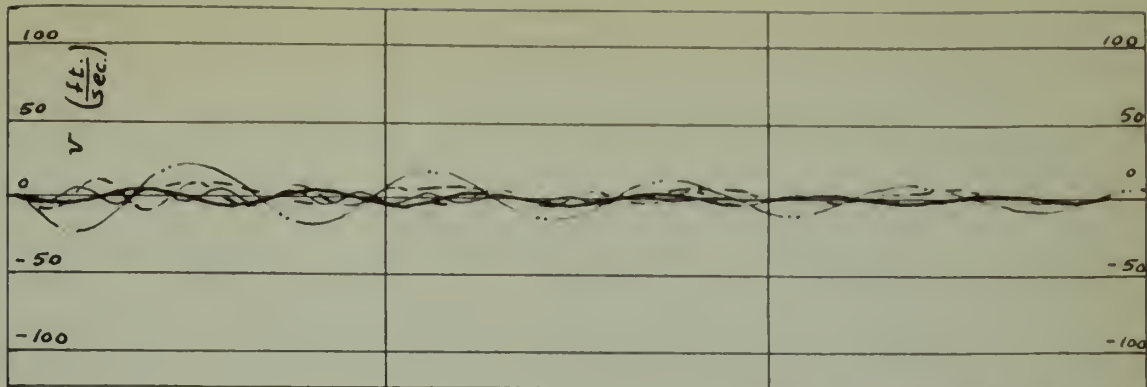


FIG. 1. Response of a single R to a sinusoidal input,  $\eta = +10^\circ, \eta = 0^\circ, \eta = -10^\circ$ . (5. = 1/50).





S.L.

Notice the scale change

H.A.

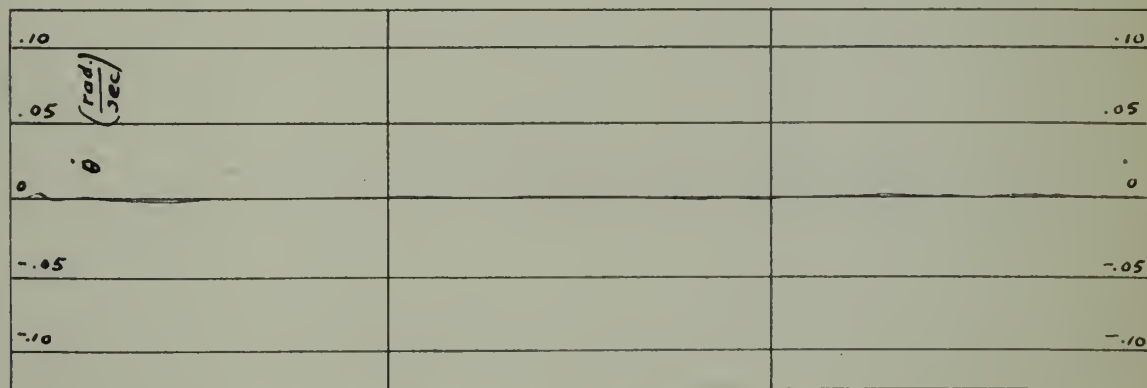
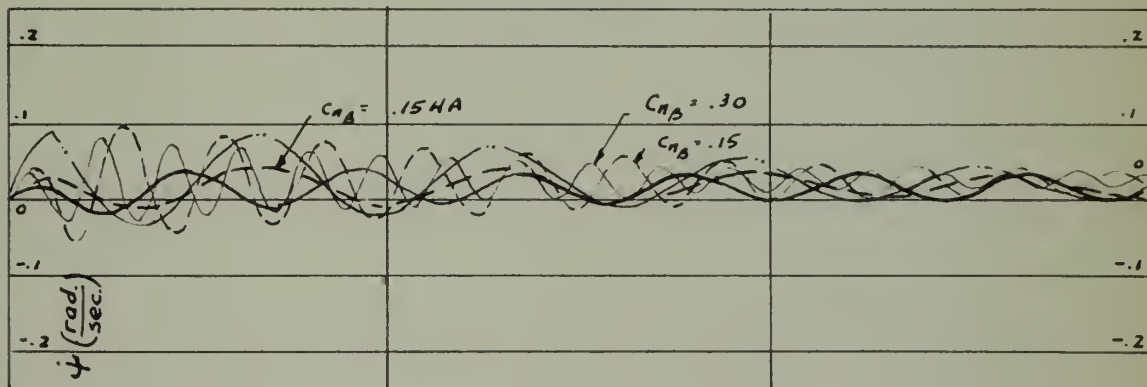
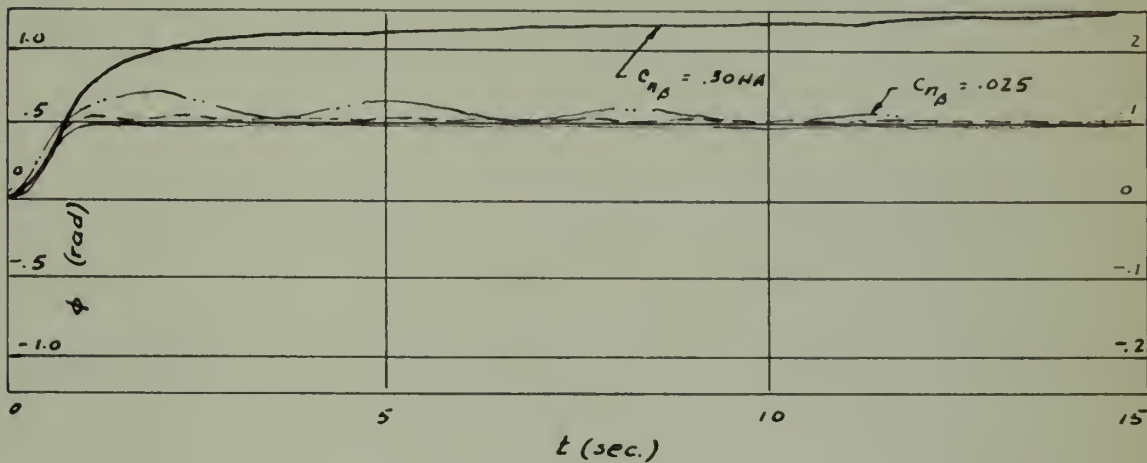
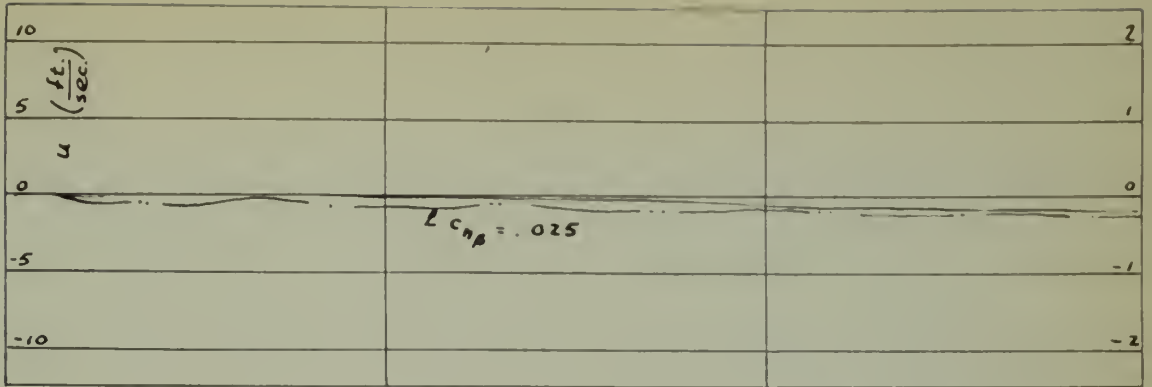


FIG. 102. RESPONSE TO A UNIT STEP INPUT,  $C_{n\beta}$  VARIED (0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95, 1.00).





S.L.

Notice the scale change H.A.

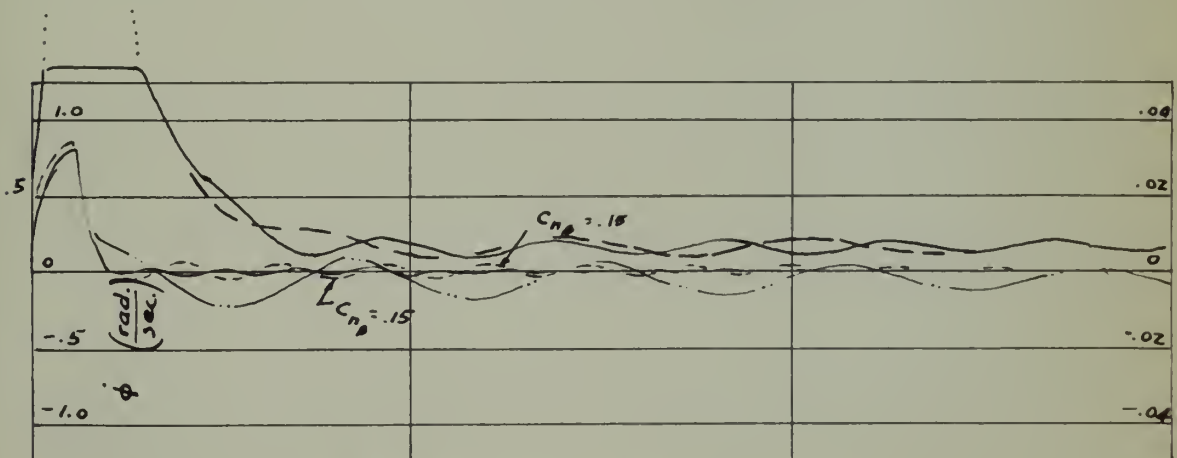
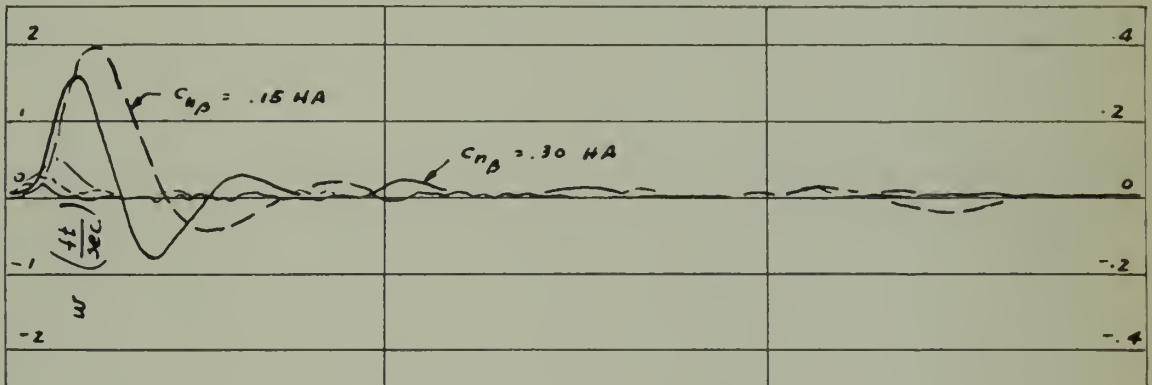
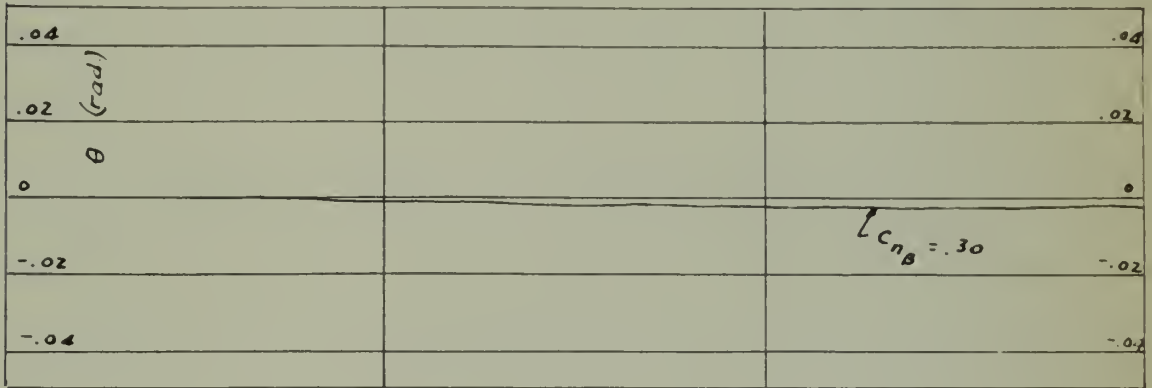
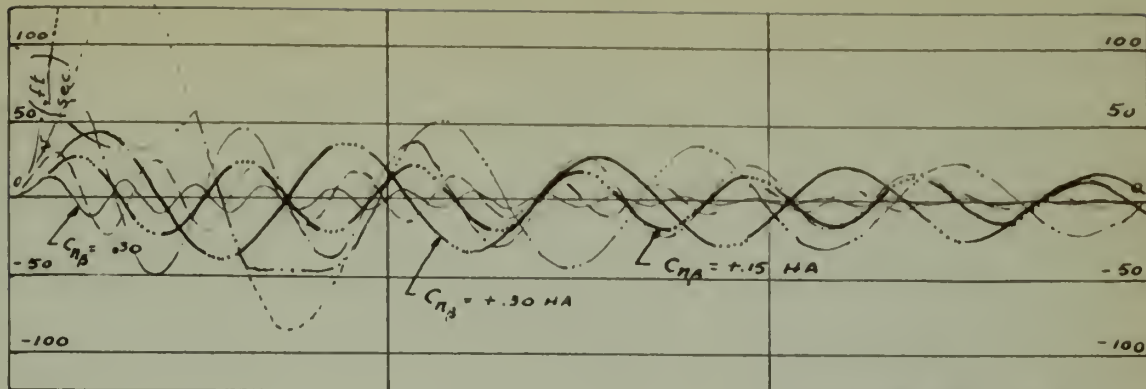


FIG. 100. R. ... T. ...  
 $c_{\eta\beta}$  ...







S.L.

Notice the scale change  
High Altitude

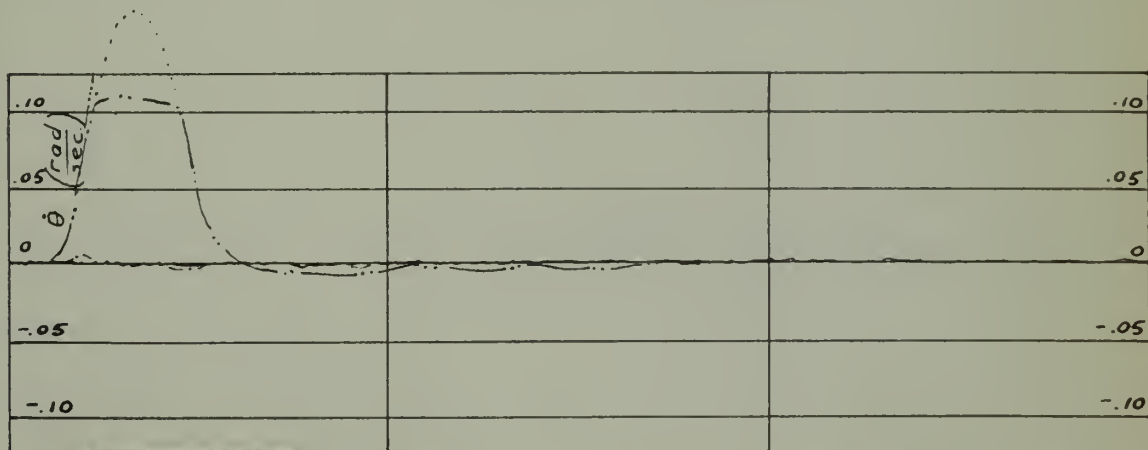
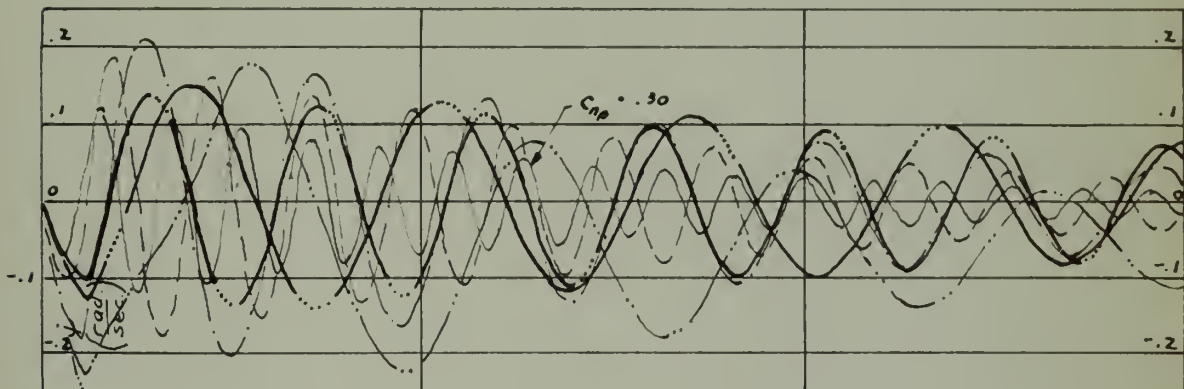
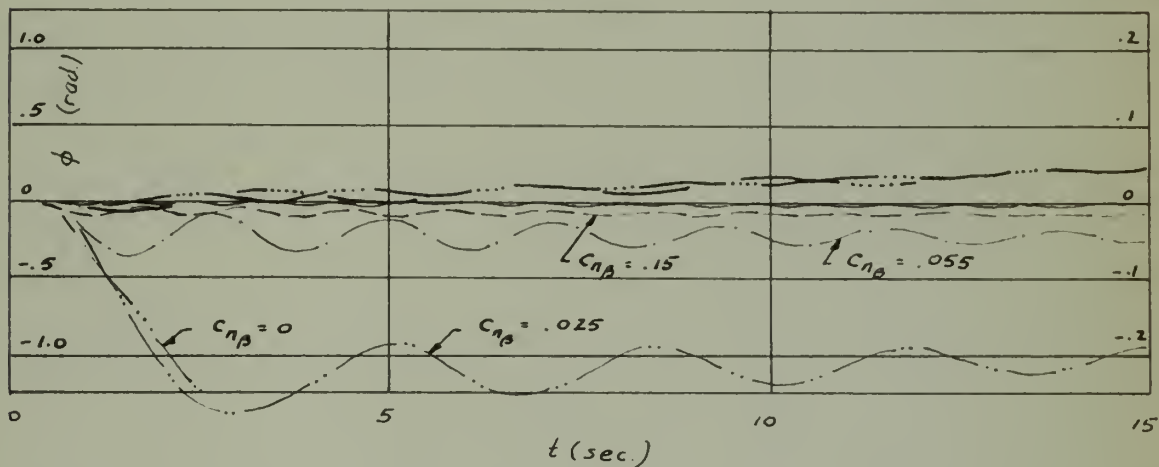
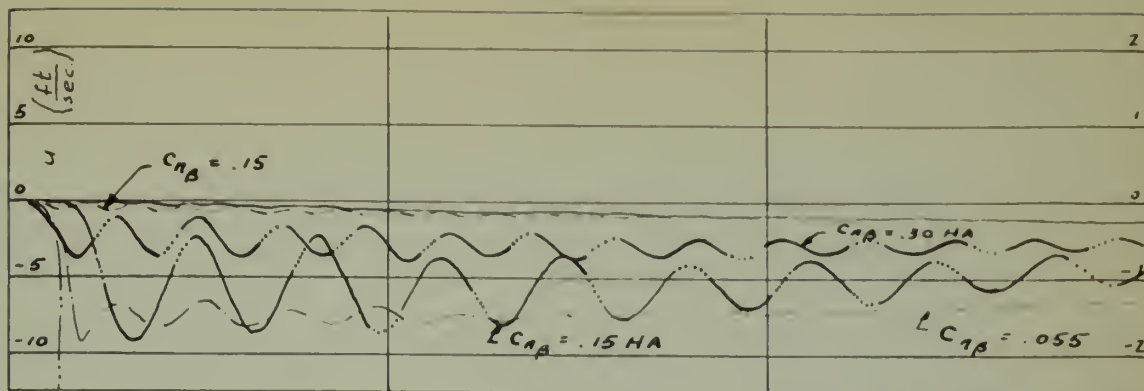


FIG. 11. Response to a step change in altitude.  
 $C_{n\beta}$  varied from 0.05 to 0.30.

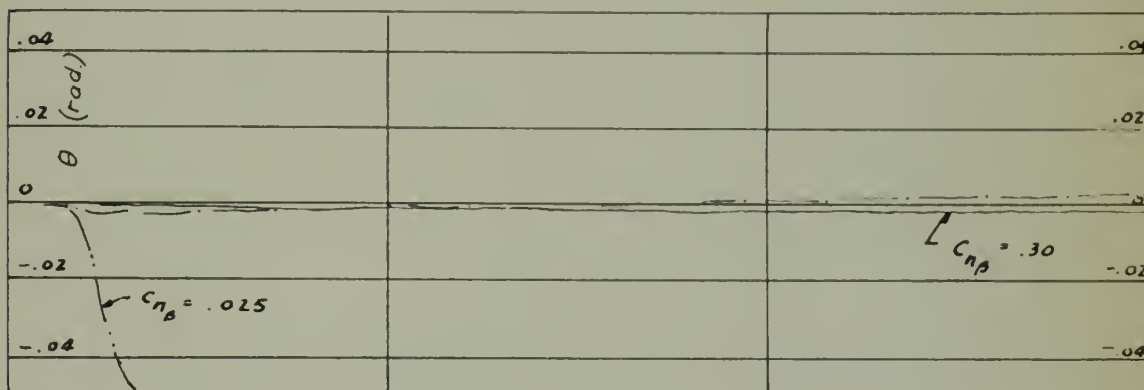




S.L.

NOTICE THE SCALE CHANGE

High Altitude



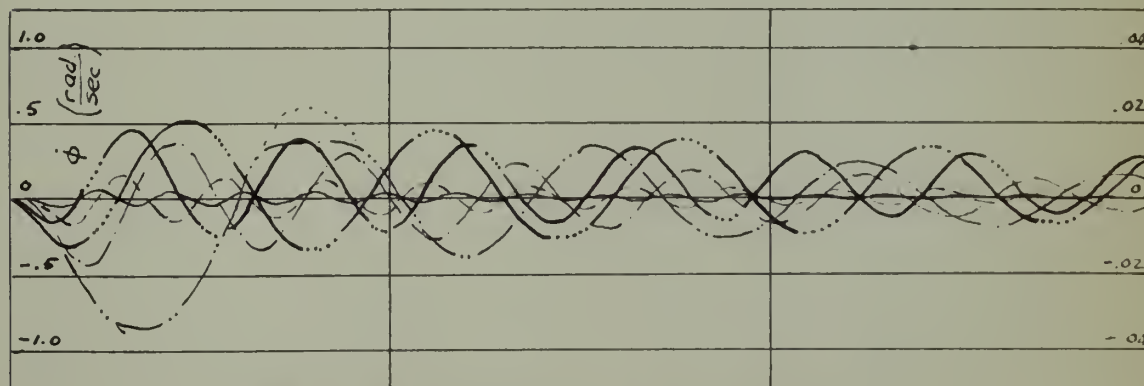
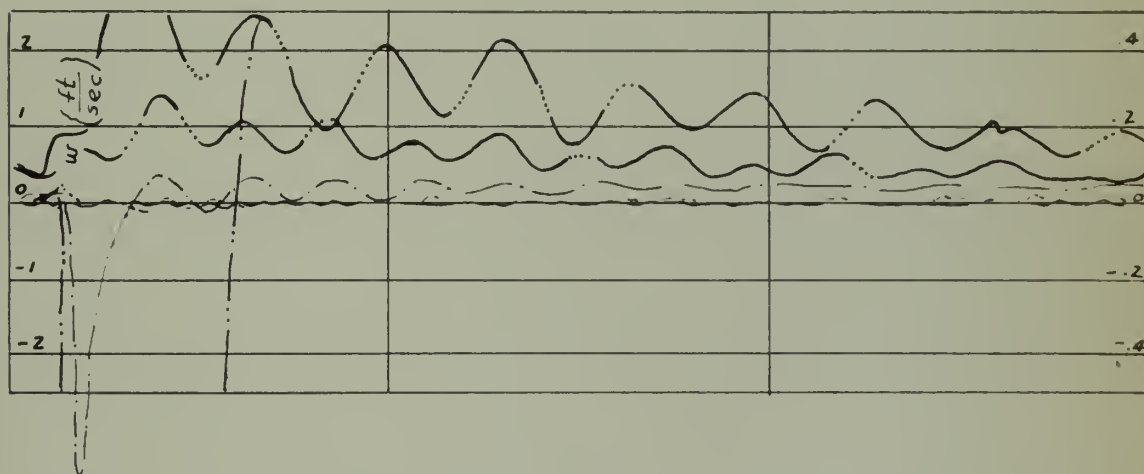
0

5

$t$  (sec)

10

15



$C_{n\beta}$



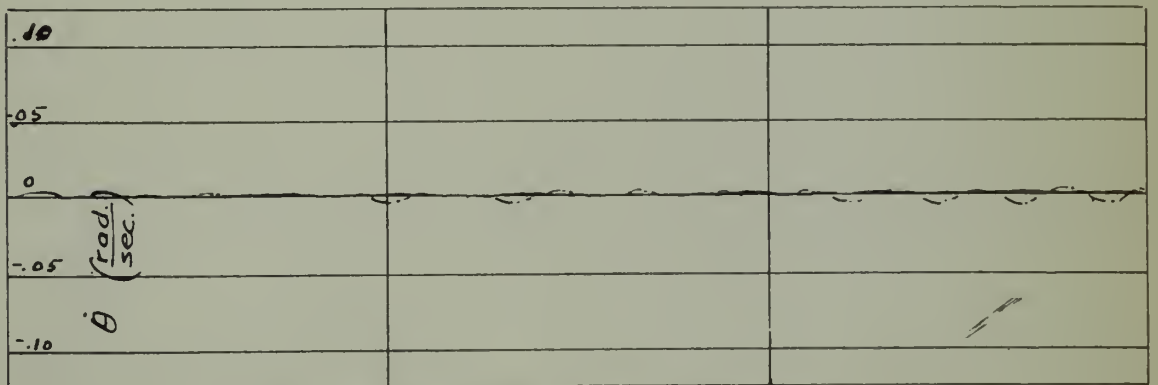
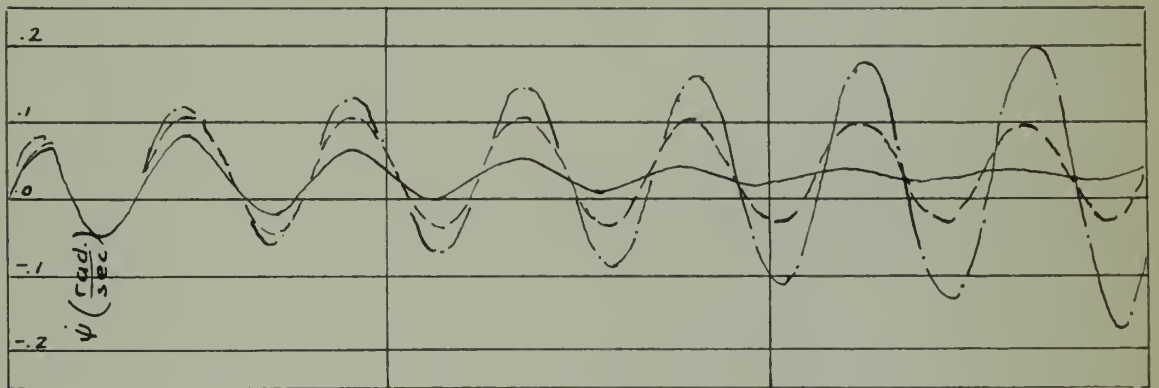
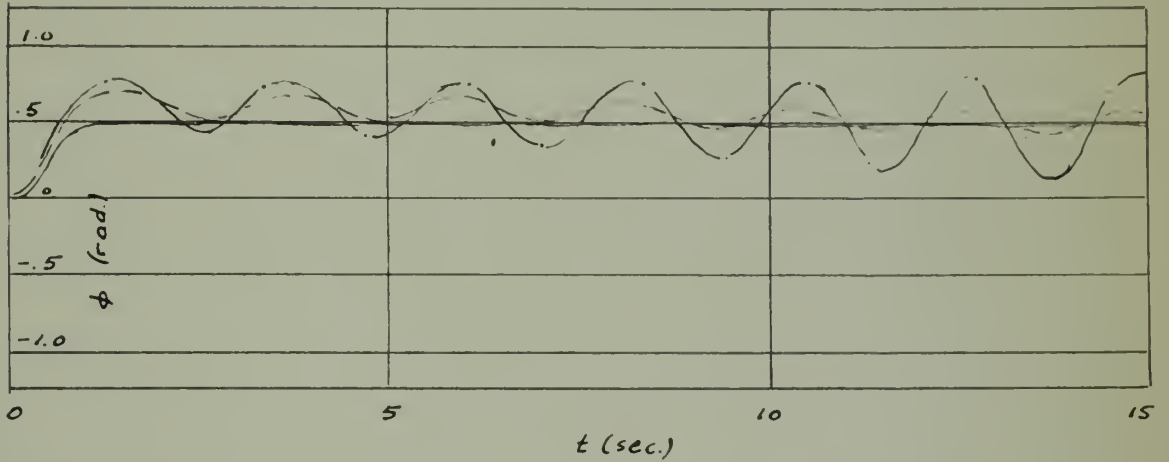
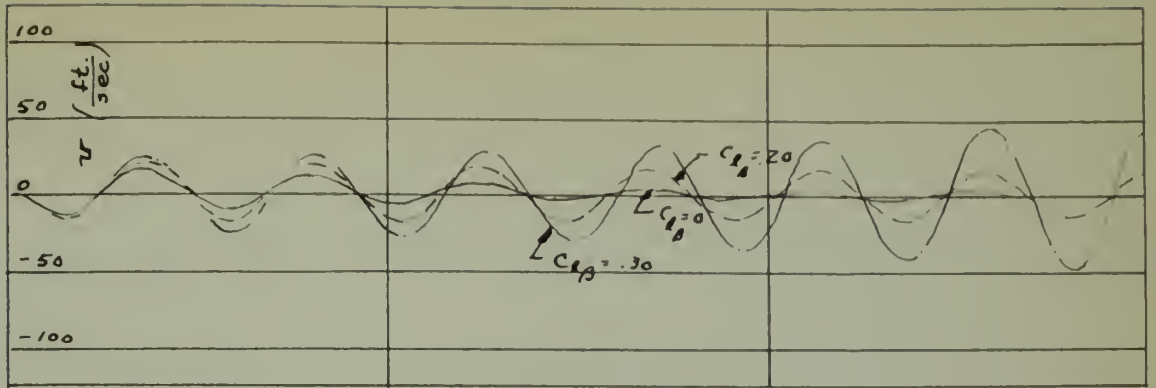


FIG. 22a. RESPONSE TO A UNIT STEP INPUT,  
 $C_{l\beta}$  VARYING FROM 0 TO 20.



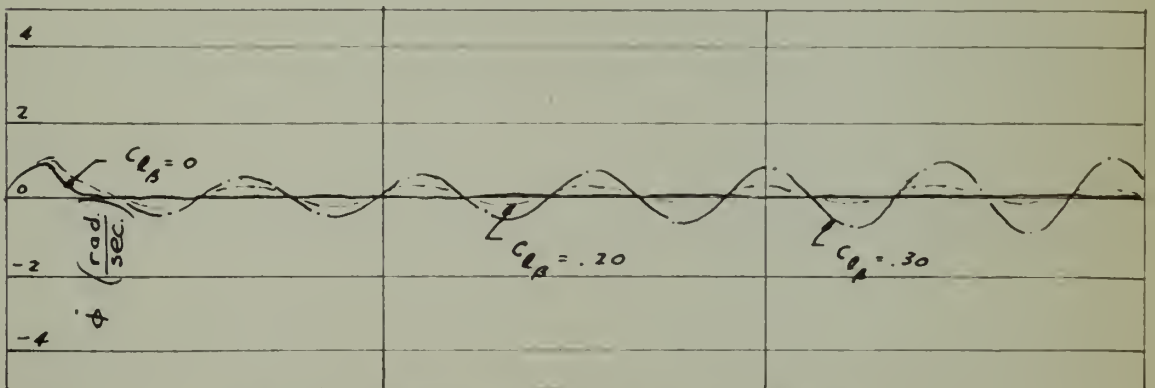
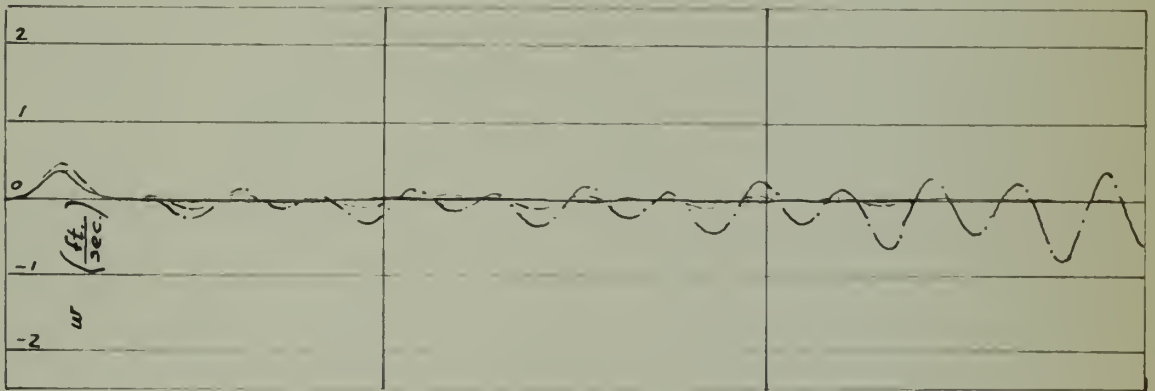
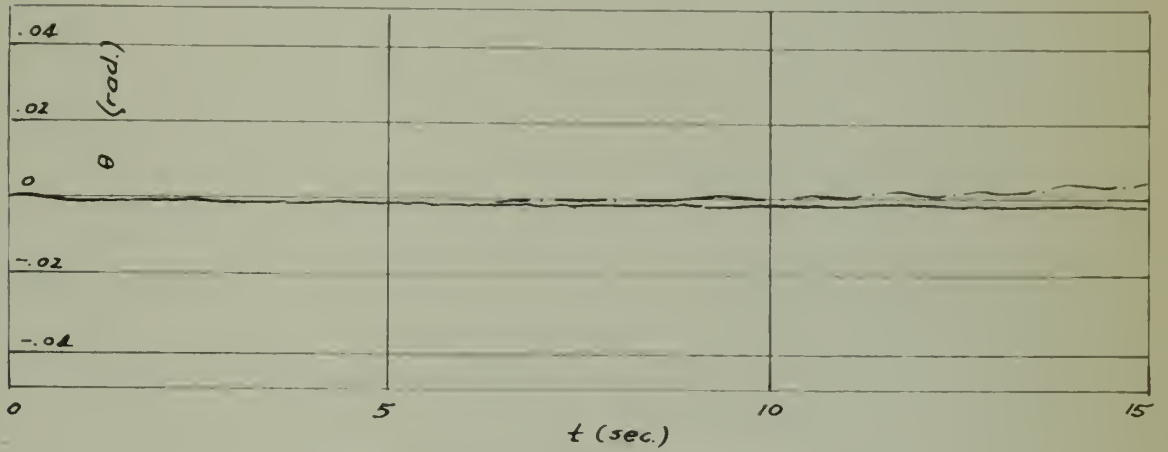
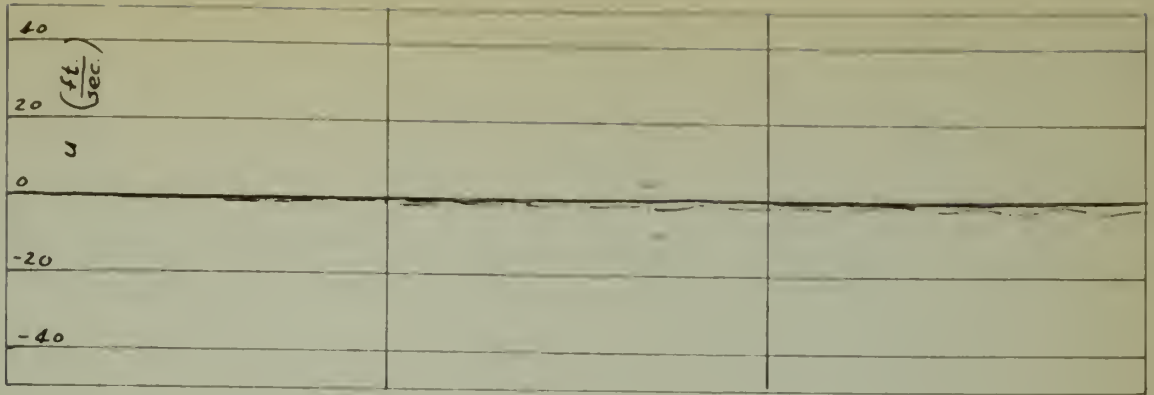
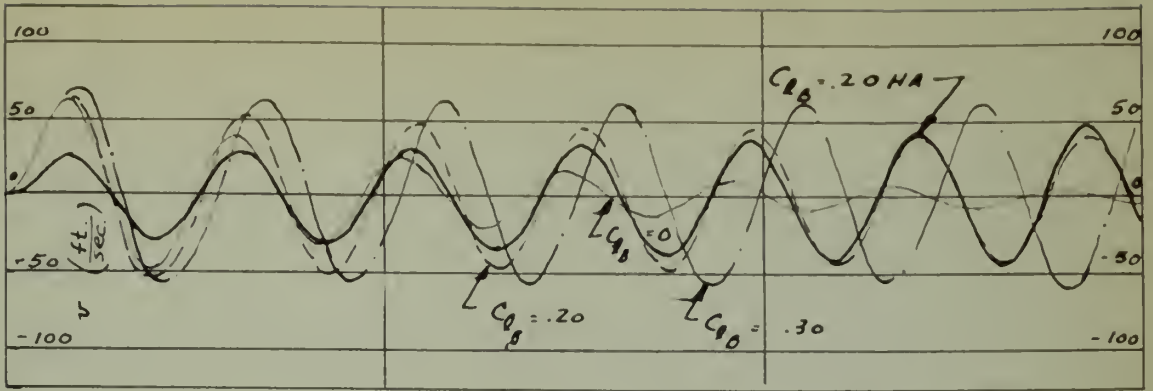


Fig. 1. Angular velocity  $\dot{\phi}$  vs. time  $t$ ,  
for various values of  $C_L$ .



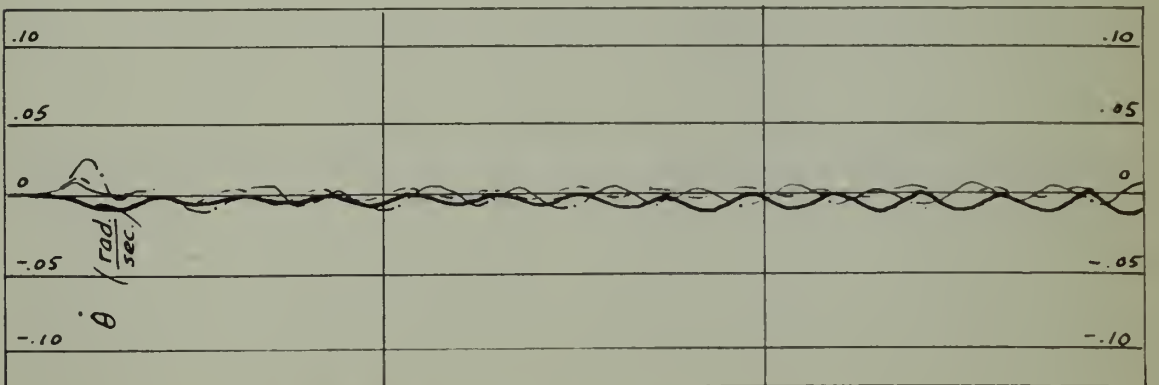
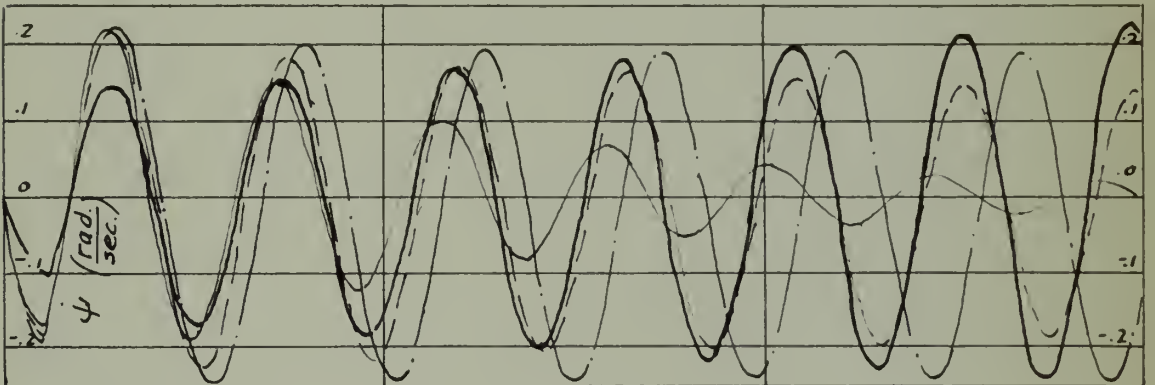
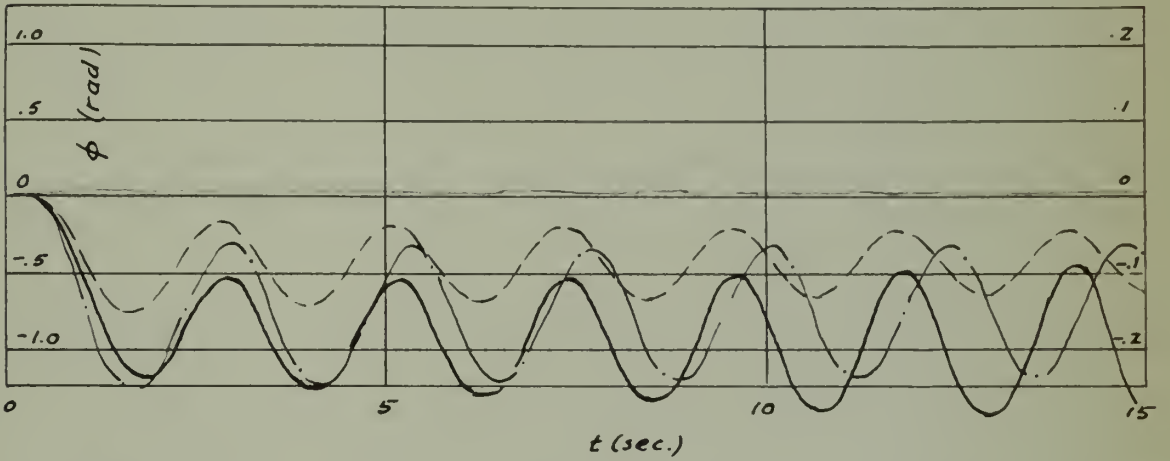




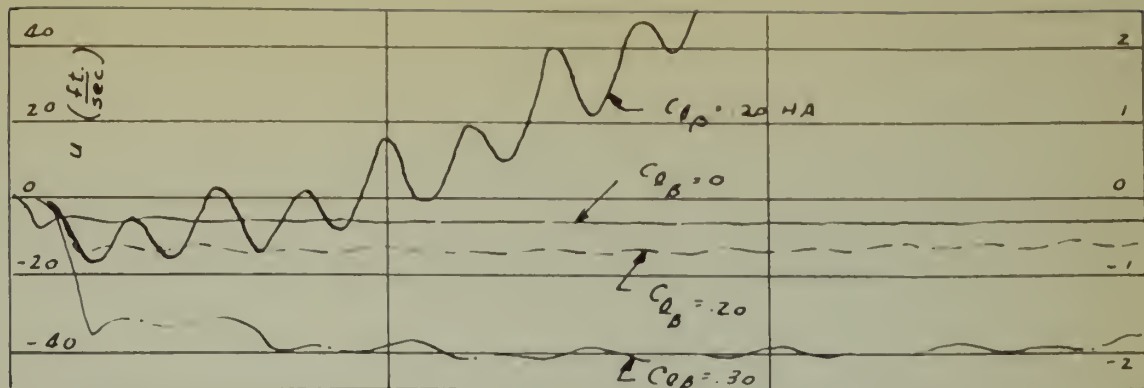
S.L.

Notice the scale change

H.A.



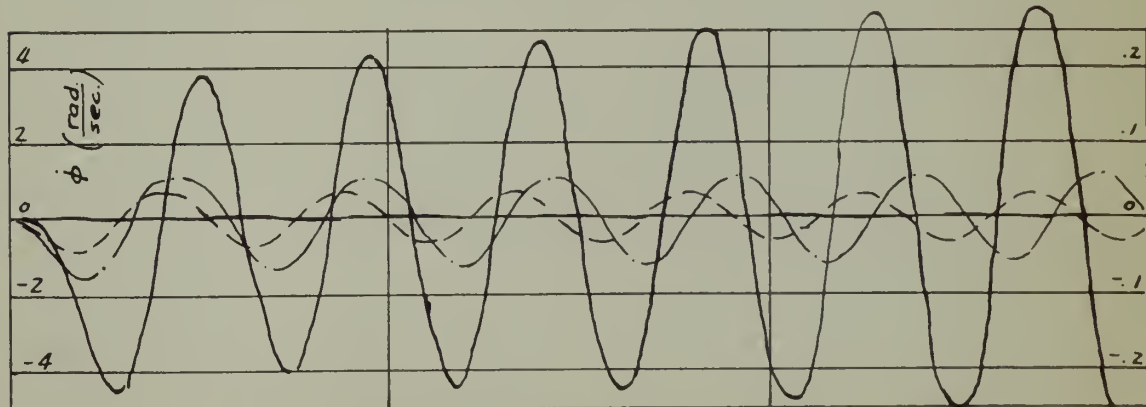
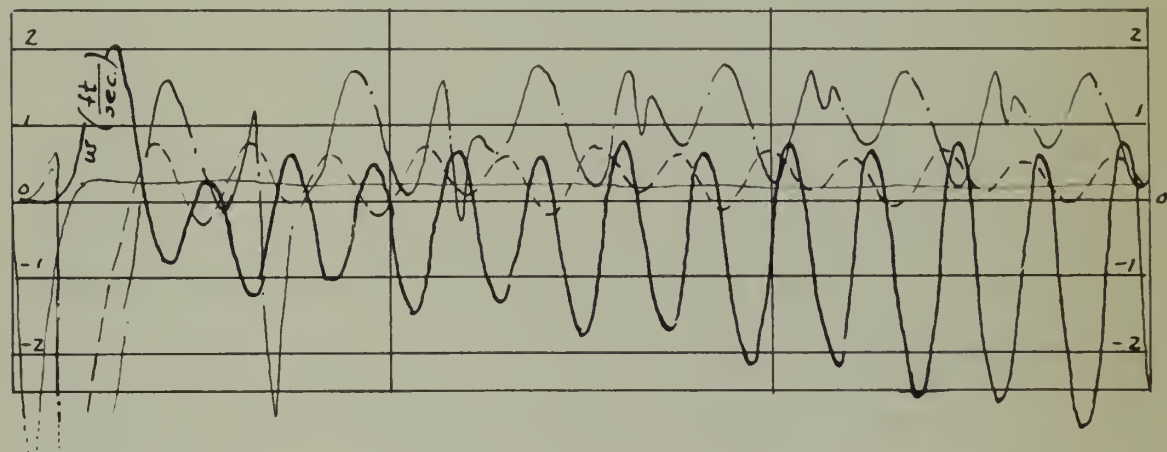
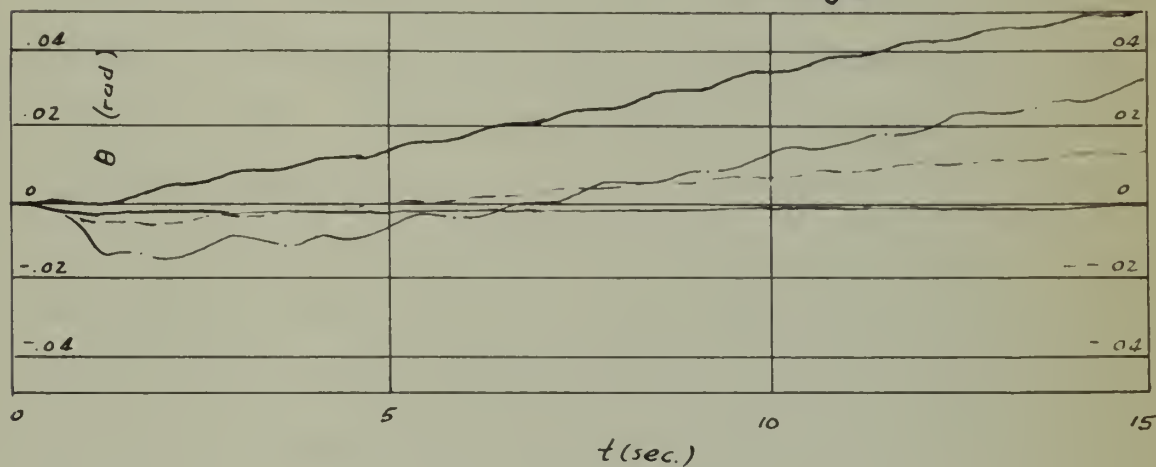




S.L.

Notice the scale change

H.A.



$C_{D\beta}$



## APPENDIX I

### Development of the Equations of Motion

The equations of motion of the airframe are written by equating the forces and moments acting on the airframe to the plane's reactions in accordance with Newton's laws. The reference axes are considered fixed in space.

The aircraft is considered to be a rigid body; aeroelastic effects are not considered. The motion is referred to right-handed Cartesian axes. The axes used and the sign conventions observed are shown in Fig. 8 and Table I.

Newton's second law states that the rate of change of momentum of a body is proportional to the force applied to the body and the rate of change of the moment of momentum is proportional to the torque applied to the body. The mass of the airplane is considered constant for the duration of the dynamic maneuver considered.

$$\begin{aligned}
 (1) \quad & \Sigma F_x = \frac{d}{dt}(mU) = m\dot{U} \\
 & \Sigma F_y = \frac{d}{dt}(mV) = m\dot{V} \\
 & \Sigma F_z = \frac{d}{dt}(mW) = m\dot{W} \\
 (2) \quad & \Sigma L = \frac{dh_x}{dt} \\
 & \Sigma M = \frac{dh_y}{dt} \\
 & \Sigma N = \frac{dh_z}{dt}
 \end{aligned}$$





Investigating the dynamics of an infinitesimal element of mass  $dm$  of the airplane referred to the Cartesian coordinate system yields equation set (3).

$$(3) \quad \begin{aligned} dh_x &= (y^2+z^2)Pdm - zxRdm - yxQdm \\ dh_y &= (z^2+x^2)Qdm - xyPdm - yzRdm \\ dh_z &= (x^2+y^2)Rdm - zxPdm - zyQdm \end{aligned}$$

For a finite mass these become

$$(4) \quad \begin{aligned} h_x &= P \int (y^2+z^2)dm - Q \int xydm - R \int xzdm \\ h_y &= Q \int (z^2+x^2)dm - R \int yzdm - P \int yx dm \\ h_z &= R \int (x^2+y^2)dm - P \int zx dm - Q \int zy dm \end{aligned}$$

The integral  $\int (y^2+z^2)dm$  is recognized as the moment of inertia  $I_x$  of the whole airplane about the x-axis. Equation set (4) can be written

$$\begin{aligned} h_x &= PI_x - QI_{xy} - RI_{xz} \\ h_y &= QI_y - RI_{yz} - PI_{xy} \\ h_z &= RI_{zz} - PI_{xz} - QI_{yz} \end{aligned}$$

The equations of motion relative to inertial axes becomes:

$$(5) \quad \begin{aligned} \Sigma F_x &= m \frac{dU}{dt} \\ \Sigma F_y &= m \frac{dV}{dt} \\ \Sigma F_z &= m \frac{dW}{dt} \end{aligned}$$

$$(6) \quad \begin{aligned} \Sigma L = \frac{dh_x}{dt} &= \dot{P}I_x + P\dot{I}_x - \dot{Q}I_{xy} - Q\dot{I}_{xy} - R\dot{I}_{xz} - R\dot{I}_{xz} \\ \Sigma M = \frac{dh_y}{dt} &= \dot{Q}I_y + Q\dot{I}_y - \dot{R}I_{yz} - R\dot{I}_{yz} - \dot{P}I_{xy} - P\dot{I}_{xy} \\ \Sigma N = \frac{dh_z}{dt} &= \dot{R}I_z + R\dot{I}_z - \dot{P}I_{xz} - P\dot{I}_{xz} - \dot{Q}I_{yz} - Q\dot{I}_{yz} \end{aligned}$$

Equations (6) are written with respect to fixed inertial axes. It is expedient to use Eulerian axes with their origin at the c.g. which are fixed in the airplane and move with it. At any instant the airplane



in a maneuver has a motion referred to the Eulerian axes and the axes have a motion referred to fixed space. The earth and its atmosphere are regarded as fixed space. The absolute acceleration of the aircraft referred to the Eulerian axes can be written

$$(7) \quad \vec{a}_{abs} = \frac{d\vec{V}_T}{dt} + \vec{\omega} \times \vec{V}_T$$

where  $\vec{V}_T$  is the total velocity.

Similarly the absolute rate of change of the moment of momentum is written

$$(8) \quad \frac{d\vec{h}_{abs}}{dt} = \frac{d\vec{h}}{dt} + \vec{\omega} \times \vec{h}$$

The first term arises from the change relative to the Eulerian axes fixed from instant to instant in the airplane and the second considers the change of the Eulerian axes relative to inertial axes fixed in space.

Moments and products of inertia measured relative to Eulerian axes are independent of time since the mass is assumed constant for the duration of a maneuver. Many of the terms in equation set (6) go to zero for this reason.

The term  $\vec{\omega} \times \vec{V}_T$  of equation (7) can be written:

$$(8a) \quad \vec{\omega} \times \vec{V}_T = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ P & Q & R \\ U & V & W \end{vmatrix}.$$

Where  $\vec{i}, \vec{j}, \vec{k}$  are unit vectors directed along the Eulerian  $X, Y, Z$  axes.

Expanding the cross product and combining with the components of the first term of (7) that equation can be written in component form as:

$$(9) \quad \begin{aligned} a_x &= \dot{U} + QW - RV \\ a_y &= \dot{V} + RU - PW \\ a_z &= \dot{W} + PV - QU \end{aligned}$$



By parallel reasoning (8) can be written:

$$\begin{aligned}
 (10) \quad \frac{dh_x}{dt} \Big|_{abs} &= \frac{dh_x}{dt} + h_z Q - h_y R = \dot{P} I_x - Q \bar{I}_{xy} - \dot{R} \bar{I}_{xz} + QR(I_z - I_y) \\
 &\quad - PQ \bar{I}_{xz} - Q^2 \bar{I}_{zy} + R^2 \bar{I}_{yz} + PR \bar{I}_{xy} \\
 \frac{dh_y}{dt} \Big|_{abs} &= \frac{dh_y}{dt} + h_x R - h_z P = \dot{Q} I_y - \dot{R} \bar{I}_{yz} - \dot{P} \bar{I}_{xy} + PR(I_x - I_z) \\
 &\quad - QR \bar{I}_{xy} - R^2 \bar{I}_{xz} + P^2 \bar{I}_{xz} + PQ \bar{I}_{zy} \\
 \frac{dh_z}{dt} \Big|_{abs} &= \frac{dh_z}{dt} + h_y P - h_x Q = \dot{R} I_z - \dot{P} \bar{I}_{xz} - \dot{Q} \bar{I}_{zy} + PQ(I_y - I_x) \\
 &\quad - PR \bar{I}_{yz} - P^2 \bar{I}_{xy} + Q^2 \bar{I}_{xy} + QR \bar{I}_{xz}
 \end{aligned}$$

With the Eulerian axes as chosen and shown in Fig. 8, the  $xz$ -plane is a plane of symmetry. There is both a positive and a negative value of  $y$  for each value of  $z$ ; consequently  $\bar{I}_{yz} = \int yz dm = 0$  and  $\bar{I}_{xy} = \int xy dm = 0$ .

The equations of motion of an aircraft referred to Eulerian axes can then be written:

$$\begin{aligned}
 (11) \quad \sum F_x &= m [\dot{U} + QW - RV] \\
 \sum F_y &= m [\dot{V} + RU - PW] \\
 \sum F_z &= m [\dot{W} + PV - QU] \\
 \sum L &= \dot{P} I_x - \dot{R} \bar{I}_{xz} + QR(I_z - I_y) - PQ \bar{I}_{xz} \\
 \sum M &= \dot{Q} I_y + PR(I_x - I_z) - R^2 \bar{I}_{xz} + P^2 \bar{I}_{xz} \\
 \sum N &= \dot{R} I_z - \dot{P} \bar{I}_{xz} + PQ(I_y - I_x) + QR \bar{I}_{xz}
 \end{aligned}$$

The left sides of the equations in set (11) are the summations of the external forces and moments applied to the airplane in flight. These external forces are gravity forces, aerodynamic forces, and thrust forces.

The gravity force may be considered to act at the center of gravity making no contribution to the summation of moments.





The resolution of the gravity force into components referred to disturbed Eulerian axes is shown in Fig. 1.

The force relations may be written

$$(12) \quad \begin{aligned} \Sigma F_x' &= m[\dot{U} + QW - RV] - X_3 \\ \Sigma F_y' &= m[\dot{V} + RU - PW] - Y_3 \\ \Sigma F_z' &= m[\dot{W} + PV - QU] - Z_3 \end{aligned}$$

by transferring the gravity terms to the right-hand side of the equation. The gravity terms  $X_3$ ,  $Y_3$ , and  $Z_3$  are expanded in Fig. 1.

The left side of all the force and moment equations now include aerodynamic and thrust forces as well as moments due to control surface deflection.

The disturbed motion of an airplane at any instant can be considered the result of disturbing the airplane from some steady flight condition. This motion may be referred to as disturbed flight and the Eulerian axes as the disturbed axes. The forces acting on the airplane under these conditions may be regarded as the sum of the force required to produce steady flight plus an increment required for the disturbance. Accordingly, each of the total instantaneous velocity components can be written as the sum of a constant velocity component during steady flight and a change caused by the disturbance:

$$(13) \quad \begin{aligned} U &= U_0 + u \\ V &= V_0 + v \\ W &= W_0 + w \end{aligned}$$





$$P = P_0 + p$$

$$Q = Q_0 + q$$

$$R = R_0 + r$$

Substituting in (12) considering that  $\frac{dU_0}{dt} = \frac{dP_0}{dt} = 0$ , etc., and using the expansion of the gravity terms from Fig. 1 after setting terms like  $\sin \psi \sin \phi = 0$ ,  $\sin \psi = \psi$  and  $\cos \theta = 1$ , the set of equations below can be written. The assumptions that the sine of an angle can be replaced by the angle, the cosine is equal to one and the product of sines can be replaced by zero produces results which, although approximate, are in good agreement with physical results according to Ref. 7. These assumptions, of course, apply to the disturbed angles.

$$\begin{aligned} (14) \quad \Sigma F'_x &= m [\dot{u} + Q_0 W_0 + W_0 q + Q_0 w - R_0 V_0 - R_0 r - V_0 R + w q - r R \\ &\quad + g \sin \theta_0 - (g \cos \theta_0 \sin \phi_0) \psi + (g \cos \theta_0 \cos \phi_0) \theta] \\ \Sigma F'_z &= m [\dot{w} + P_0 V_0 + P_0 r + V_0 p - Q_0 U_0 - Q_0 u - U_0 q - q u + p r \\ &\quad + (g \sin \theta_0) \psi + (g \cos \theta_0 \sin \phi_0) \phi - (g \cos \theta_0 \cos \phi_0) \theta] \\ \Sigma F'_y &= m [\dot{v} + U_0 R_0 + U_0 r + R_0 u - P_0 W_0 - P_0 w - W_0 p - w p + r u \\ &\quad - (g \sin \theta_0) \psi - g \cos \theta_0 \sin \phi_0 - (g \cos \theta_0 \cos \phi_0) \phi] \\ \Sigma L &= \dot{p} I_x - \dot{r} I_{xz} + (Q_0 R_0 + Q_0 r + R_0 q + q r)(I_z - I_y) \\ &\quad - (P_0 Q_0 + P_0 q + Q_0 p + p q) I_{xz} \\ \Sigma M &= \dot{q} I_y + (P_0 R_0 + P_0 r + R_0 p + p r)(I_x - I_z) - (R_0^2 + 2 R_0 r + r^2) I_{xz} \\ &\quad + (P_0^2 + 2 P_0 p + p^2) I_{xz} \\ \Sigma N &= \dot{r} I_z - \dot{p} I_{xz} + (P_0 Q_0 + P_0 q + Q_0 p + p q)(I_y - I_x) + (Q_0 R_0 + Q_0 r + R_0 q + r q) I_{xz} \end{aligned}$$

The aerodynamic forces and moments acting on an airplane, the left side of the above equations, are functions of the flight conditions and of the deflection of the control surfaces.



Each of the forces ( $X, Y$ , and  $Z$ ) and the moments ( $L, M$ , and  $N$ ) can be expressed for small oscillations by expanding by MacLaurin's theorem neglecting second order terms to obtain:

$$\begin{aligned}
 X &= X_0 + u \frac{\partial X}{\partial u} + q \frac{\partial X}{\partial q} + w \frac{\partial X}{\partial w} + \dot{w} \frac{\partial X}{\partial \dot{w}} \dots \\
 \text{where } \frac{\partial X}{\partial u}, \frac{\partial X}{\partial q}, \text{ etc. are partial derivatives of } X \\
 &\text{with respect to } u, q, \text{ etc. Exactly analogous} \\
 &\text{expansions are made for } Y, Z, L, M, \text{ and } N. \text{ These} \\
 &\text{forces and moments can be expressed:} \\
 (15) \quad X &= X_0 + \frac{\partial X}{\partial u} u + \frac{\partial X}{\partial q} q + \frac{\partial X}{\partial w} w + \frac{\partial X}{\partial \dot{w}} \dot{w} \dots \\
 Y &= Y_0 + \frac{\partial Y}{\partial r} r + \frac{\partial Y}{\partial v} v + \frac{\partial Y}{\partial p} p + \frac{\partial Y}{\partial \delta_a} \delta_a + \frac{\partial Y}{\partial \delta_r} \delta_r \dots \\
 Z &= Z_0 + \frac{\partial Z}{\partial u} u + \frac{\partial Z}{\partial q} q + \frac{\partial Z}{\partial w} w + \frac{\partial Z}{\partial \delta_e} \delta_e \dots \\
 L &= L_0 + \frac{\partial L}{\partial r} r + \frac{\partial L}{\partial v} v + \frac{\partial L}{\partial p} p + \frac{\partial L}{\partial \delta_a} \delta_a + \frac{\partial L}{\partial \delta_r} \delta_r \dots \\
 M &= M_0 + \frac{\partial M}{\partial u} u + \frac{\partial M}{\partial q} q + \frac{\partial M}{\partial w} w + \frac{\partial M}{\partial \delta_e} \delta_e \dots \\
 N &= N_0 + \frac{\partial N}{\partial v} v + \frac{\partial N}{\partial r} r + \frac{\partial N}{\partial p} p + \frac{\partial N}{\partial \delta_r} \delta_r + \frac{\partial N}{\partial \delta_a} \delta_a \dots
 \end{aligned}$$

Each of the terms in (15) has a physical significance.  $X_0, Y_0$ , and  $Z_0$ ,  $L_0, M_0$ , and  $N_0$  are the forces and moments acting along and about the  $x, y$ , and  $z$  axes while the plane is in steady flight. The terms similar to  $\frac{\partial X}{\partial u} u$  express the change in the given force or moment caused by the given disturbance quantity. The coefficients of the form  $\frac{\partial L}{\partial p}$  are the basic dimensional stability derivatives. A complete discussion of stability derivatives may be found in Refs. (1), (2), (3), (7), (8), and (9).



Thrust effects are part of the left sides of equation set (14). The equations for steady flight condition become:

$$(16) \quad \begin{aligned} X_o &= T_o \cos \xi \\ Z_o &= -T_o \sin \xi \\ M_o &= T_o z_j \end{aligned}$$

Since the Eulerian axes remain fixed with reference to the airplane during a disturbance, the thrust components relative to disturbed axes become:

$$(17) \quad \begin{aligned} X &= T_i \cos \xi \\ Z &= -T_i \sin \xi \\ M &= T_i z_j \end{aligned}$$

where  $T_i$ , the thrust during the disturbance, is  $T_o + \Delta T$ .  $\Delta T$  can be considered

$$(18) \quad \Delta T = \frac{\partial T}{\partial u} u + \frac{\partial T}{\partial \delta_{rpm}} \delta_{rpm}$$

and

$$(19) \quad \begin{aligned} X &= T_o \cos \xi + (\cos \xi) \frac{\partial T}{\partial u} u + (\cos \xi) \frac{\partial T}{\partial \delta_{rpm}} \delta_{rpm} \\ Z &= -T_o \sin \xi - (\sin \xi) \frac{\partial T}{\partial u} u - (\sin \xi) \frac{\partial T}{\partial \delta_{rpm}} \delta_{rpm} \\ M &= T_o z_j + z_j \frac{\partial T}{\partial u} u + z_j \frac{\partial T}{\partial \delta_{rpm}} \delta_{rpm} \end{aligned}$$

The equations for steady flight are found by substituting the expressions obtained above for the thrust and the aerodynamic forces (and moments) into the left sides of equation set (14) and setting the disturbed quantities equal to zero:

$$(20) \quad \begin{aligned} X_o - W \sin \theta_e + T_o \cos \xi &= 0 \\ Y_o + 0 + 0 &= 0 \\ Z_o + W \cos \theta_o - T_o \sin \xi &= 0 \end{aligned}$$







$$L_0 + 0 + 0 = 0$$

$$M_0 + 0 + T_0 z_j = 0$$

$$N_0 + 0 + 0 = 0$$

If now the complete equations for the disturbed airplane are written with both steady flight and disturbed terms they will be of the form:

Aerodynamic forces + thrust forces = inertia force = gravity force

Equation set (20) can then be subtracted from the complete equations of motion for the disturbed airplane yielding:

$$\begin{aligned}
 (21) \quad & \frac{\partial X}{\partial u} u + \frac{\partial X}{\partial q} q + \frac{\partial X}{\partial w} w + \frac{\partial X}{\partial \dot{w}} \dot{w} + (\cos \xi) \frac{\partial T}{\partial u} u + (\cos \xi) \frac{\partial T}{\partial \delta_{rpm}} \delta_{rpm} \\
 & = m [\dot{u} + Q_0 W_0 + W_0 q + Q_0 w + w q - R_0 Y_0 - R_0 v - V_0 r - r R \\
 & \quad - (g \cos \theta_0 \sin \phi_0) \psi + (g \cos \theta_0 \cos \phi_0) \theta] \\
 & \frac{\partial Y}{\partial r} r + \frac{\partial Y}{\partial p} p + \frac{\partial Y}{\partial \delta_a} \delta_a + \frac{\partial Y}{\partial \delta_r} \delta_r = m [\dot{v} + U_0 R_0 + U_0 r + R_0 u + u R \\
 & \quad - P_0 W_0 - P_0 w - W_0 p - w p - (g \sin \theta_0) \psi - g \cos \theta_0 \sin \phi_0 - (g \cos \theta_0 \cos \phi_0) \phi] \\
 & \frac{\partial Z}{\partial u} u + \frac{\partial Z}{\partial q} q + \frac{\partial Z}{\partial w} w + \frac{\partial Z}{\partial \delta_e} \delta_e - (\sin \xi) \frac{\partial T}{\partial u} u - (\sin \xi) \frac{\partial T}{\partial \delta_{rpm}} \delta_{rpm} \\
 & = m [\dot{w} + P_0 V_0 + P_0 v + V_0 p + p v - Q_0 U_0 - Q_0 u - U_0 q - q u \\
 & \quad + (g \sin \theta_0) \theta + (g \cos \theta_0 \sin \phi_0) \phi - (g \cos \theta_0 \cos \phi_0) \psi] \\
 & \frac{\partial L}{\partial r} r + \frac{\partial L}{\partial v} v + \frac{\partial L}{\partial p} p + \frac{\partial L}{\partial \delta_a} \delta_a + \frac{\partial L}{\partial \delta_r} \delta_r \\
 & = \dot{p} I_x - \dot{r} I_{xz} + (Q_0 R_0 + Q_0 r + R_0 q + q r) (I_z - I_y) \\
 & \quad - (P_0 Q_0 + P_0 q + Q_0 p + p q) I_{xz} \\
 & \frac{\partial M}{\partial u} u + \frac{\partial M}{\partial q} q + \frac{\partial M}{\partial w} w + \frac{\partial M}{\partial \delta_e} \delta_e + z_j \frac{\partial T}{\partial u} u + z_j \frac{\partial T}{\partial \delta_{rpm}} \delta_{rpm} \\
 & = \dot{q} I_y + (P_0 R_0 + P_0 r + R_0 p + p r) (I_x - I_z) \\
 & \quad - (R_0^2 + z R_0 r + r^2) I_{xz} + (P_0^2 + z P_0 p + p^2) I_{xz} \\
 & \frac{\partial N}{\partial v} v + \frac{\partial N}{\partial r} r + \frac{\partial N}{\partial p} p + \frac{\partial N}{\partial \delta_r} \delta_r + \frac{\partial N}{\partial \delta_a} \delta_a \\
 & = \dot{r} I_{zz} - \dot{p} I_{xz} + (P_0 Q_0 + P_0 q + Q_0 p + p q) (I_y - I_x) \\
 & \quad + (Q_0 R_0 + Q_0 r + R_0 q + q r) I_{xz}
 \end{aligned}$$



Dividing the force equations by  $m$  and the moment equations by the appropriate moment of inertia yields terms of the form:

$$\frac{1}{m} \frac{\partial X}{\partial u} u \quad \text{and} \quad \frac{1}{I_x} \frac{\partial L}{\partial r} r$$

Replacing  $\frac{1}{m} \frac{\partial X}{\partial u} u$  by  $X_u$  and  $\frac{1}{I_x} \frac{\partial L}{\partial r} r$  by  $L_r$  simplifies the notation. These quantities are called dimensional stability derivatives. A list of these stability derivatives in terms of the dimensionless forms appears as Tables III and IV. The flight path angle  $\gamma_0$  replaces  $\theta_0$ . The equations in final form can be written:

$$(22) \quad X_u u + X_q q + X_w w + X_{\dot{w}} \dot{w} + \cos \xi T_u u + \cos \xi T_{\delta_{rpm}} \delta_{rpm} \\ = \dot{u} + W_0 q + Q_0 w + w q - R_0 V_0 - R_0 r - V_0 r - r R \\ - (g \cos \gamma_0 \sin \phi_0) \psi$$

$$Y_{rr} r + Y_v v + Y_p p + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r = \dot{v} + U_0 r - W_0 p - w p + U_0 R_0 + U_0 r \\ + R_0 u + u R - P_0 w - (g \sin \gamma_0) \psi - g \sin \gamma_0 \sin \phi_0 - (g \cos \gamma_0 \cos \phi_0) \phi$$

$$Z_{uu} u + Z_q q + Z_w w + Z_{\delta_e} \delta_e - \sin \xi T_u u - T_{\delta_{rpm}} \delta_{rpm} = \\ \dot{w} + P_0 V_0 + P_0 r + V_0 p - Q_0 U_0 - Q_0 u - V_0 q - q u + p v \\ - (g \sin \theta_0) \theta + (g \cos \theta_0 \sin \phi_0) \phi + (g \cos \theta_0 \cos \phi_0) - g \cos \gamma_0$$

$$\dot{p} - \dot{r} \frac{I_{xz}}{I_x} + (Q_0 R_0 + Q_0 r + R_0 q + q r) \left( \frac{I_z - I_y}{I_x} \right) - (P_0 Q_0 + P_0 q + Q_0 p + p q) \frac{I_{xz}}{I_x} \\ = L_r r + L_v v + L_p p + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r$$

$$\dot{q} + (P_0 R_0 + P_0 r + R_0 p + p r) \left( \frac{I_x - I_z}{I_y} \right) - (R_0^2 + 2R_0 r + r^2) \frac{I_{xz}}{I_y} \\ + (P_0^2 + 2P_0 p + p^2) \frac{I_{xz}}{I_y} = M_u u + M_q q + M_w w + M_{\delta_e} \delta_e$$

$$\dot{r} - \dot{p} \frac{I_{xz}}{I_z} + (P_0 Q_0 + P_0 q + Q_0 p + p q) \left( \frac{I_y - I_x}{I_z} \right) + (Q_0 R_0 + Q_0 r + R_0 q + q r) \frac{I_{xz}}{I_z} \\ = N_v v + N_r r + N_p p + N_{\delta_r} \delta_r + N_{\delta_a} \delta_a$$



## APPENDIX II

### Adaptation of the Equations of Motion to the Analogue Computer

The dynamic equations of motion of the airplane were developed in Appendix I. These are summarized below:

$$\begin{aligned} (\text{II-1}) \quad & X_u u + X_q q + X_w w + X_{\dot{w}} \dot{w} + \cos \xi T_u u + \cos \xi T_{\delta_{rpm}} \delta_{rpm} \\ & = \dot{u} + W_0 q + Q_0 w + w q - R_0 V_0 - R_0 v - V_0 r - v r \\ & - (g \cos \gamma_0 \sin \phi_0) \psi \end{aligned}$$

$$\begin{aligned} (\text{II-2}) \quad & Y_r r + Y_v v + Y_p p + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r = \dot{v} + U_0 r - W_0 p \\ & - w p + U_0 R_0 + U_0 r + R_0 u + u r - P_0 w \\ & - (g \sin \gamma_0) \psi - g \sin \gamma_0 \sin \phi_0 - (g \cos \gamma_0 \cos \phi_0) \phi \end{aligned}$$

$$\begin{aligned} (\text{II-3}) \quad & Z_u u + Z_q q + Z_w w + Z_{\delta_z} \delta_z - \sin \xi T_u u - T_{\delta_{rpm}} \delta_{rpm} = \\ & \dot{w} + P_0 V_0 + P_0 v + V_0 p - Q_0 U_0 - Q_0 u - V_0 q - q u + p v \\ & - (g \sin \theta_0) \theta + (g \cos \theta_0 \sin \phi_0) \phi + (g \cos \theta_0 \cos \phi_0) \psi - g \cos \gamma_0 \end{aligned}$$

$$\begin{aligned} (\text{II-4}) \quad & \dot{p} - \dot{r} \frac{I_{xz}}{I_x} + (Q_0 R_0 + Q_0 r - R_0 q + q r) \left( \frac{I_z - I_y}{I_x} \right) \\ & - (P_0 Q_0 + P_0 q + Q_0 p + p q) \frac{I_{xz}}{I_x} \\ & L_r r + L_v v + L_p p + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r \end{aligned}$$

$$\begin{aligned} (\text{II-5}) \quad & \dot{q} + (P_0 R_0 + P_0 r + R_0 p + p r) \frac{(I_x - I_z)}{I_y} - (R_0^2 + 2 R_0 r + r^2) \frac{I_{xz}}{I_y} \\ & + (P_0^2 + 2 P_0 p + p^2) \frac{I_{xz}}{I_y} = M_u u + M_q q + M_w w + M_{\delta_z} \delta_z \end{aligned}$$

$$\begin{aligned} (\text{II-6}) \quad & \dot{r} - \dot{p} \frac{I_{xz}}{I_z} + (P_0 Q_0 + P_0 q + Q_0 p + p q) \frac{(I_y - I_x)}{I_z} \\ & + Q_0 R_0 + Q_0 r + R_0 q + q r \frac{I_{xz}}{I_z} - N_v v + N_r r + N_p p + N_{\delta_r} \delta_r + N_{\delta_a} \delta_a \end{aligned}$$

These equations are general and allow initial velocities, angular or linear, with respect to any axis. Additional control terms, such as flap deflection may be included by adding the product of



the desired deflection and the appropriate stability derivative. Any attitude of the airplane is also permitted.

In order to fit the two analogue computers available certain assumptions were necessary. The plane was assumed to be in horizontal level flight with no initial velocities except  $U_0$ , forward velocity along the  $x$ -axis. Stability axes are chosen with the undisturbed  $x$ -axis aligned with the relative wind and the  $y$ -axis a principal axis. The product of inertia will be different from zero. The terms which describe power effects are permitted to go to zero by assuming gliding flight since these power terms are not essential to the dynamic effects which are studied. They merely add complexity.

Solving the resultant equations in terms of the highest derivative places them in the form required for analogue computer solution:

$$(II-7) \quad \dot{u} = X_u u + X_q q + X_w w + X_{\dot{w}} \dot{w} + r r - w q - g \theta$$

$$(II-8) \quad \dot{v} = Y_r r + Y_v v + Y_p p + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r + w p - r u + g \phi$$

$$(II-9) \quad \dot{w} = Z_u u + Z_q q + Z_w w + Z_{\delta_e} \delta_e + q u - p v$$

$$(II-10) \quad \ddot{\psi} = \dot{r} = N_r r + N_v v + N_p p + N_{\delta_r} \delta_r + N_{\delta_a} \delta_a + \dot{p} \frac{I_{xz}}{I_z} - p q \left( \frac{I_y - I_x}{I_z} \right) - q r \frac{I_{xz}}{I_z}$$

$$(II-11) \quad \ddot{\phi} = \dot{p} = L_r r + L_v v + L_p p + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r + \dot{r} \frac{I_{xz}}{I_x} - q r \left( \frac{I_z - I_y}{I_x} \right) + p q \frac{I_{xz}}{I_x}$$

$$(II-12) \quad \ddot{\theta} = \dot{q} = M_u u + M_q q + M_w w + M_{\delta_e} \delta_e + M_{\dot{w}} \dot{w} - p r \left( \frac{I_x - I_z}{I_y} \right) + r^2 \frac{I_{xz}}{I_y} - p^2 \frac{I_{xz}}{I_y}$$





The assumptions made simplify the equations sufficiently to permit use of the analogue computers. The inertial coupling terms remain. These products are handled by the servo-multipliers and permit the airframe studied to move with six degrees of freedom.

The dimensional stability derivatives are obtained from Table V, substituting these values in the equations above yields the analogue computer equations for the basic airframe:

#### Longitudinal Equations

$$(II-13) \quad \dot{u} = -.0151 u + .0269 w + v r - w q - 32.2 \theta$$

$$(II-14) \quad \dot{w} = -.130 u + 11.55 q - 2.70 w + 61.3 \delta_e \\ + 670 q + q u - p v$$

$$(II-15) \quad \dot{q} = -.000282 u - 16.86 q - .034 w + 76.8 \delta_q - .00184 \dot{w} + .888 p r + .05 \dot{r} - .05 p^2$$

#### Lateral Equations

$$(II-16) \quad \dot{v} = +1.81 r - .173 v - .302 p + 22.4 \delta_r \\ - 670 r + w p - r u + 32.2 \phi$$

$$(II-17) \quad \dot{p} = +.1785 r - .054 v - 6.53 p + 62.5 \delta_a + 2.34 \delta_r \\ + .0755 \dot{r} - .83 q r + .0755 p q$$

$$(II-18) \quad \dot{r} = +.0125 v - .190 r + .0508 p - 5.46 \delta_r \\ + 1.21 \delta_a + .0323 \dot{p} - .218 p q - .0323 q r$$

These equations of motion are entered directly in the analogue computers. The dimensional form of the stability derivatives permits the use of real time, that is, one second of machine time is equal to one second of real time. The coefficients, which are the dimensional stability derivatives, are entered as



potentiometer settings either directly or, if they are greater than one, as a fraction which is subsequently amplified to full value. The potentiometer settings along with the parameters they represent are presented in Tables VI and VII.

The stability derivatives which represent the change in a variable due to a control movement are based on a one radian control displacement. The product of the control movement and the corresponding stability derivative represents the forcing function which disturbs the airframe from its level flight equilibrium. Since the equations are entered directly with no machine scaling there is a one to one correspondence between machine units of 100 volts and unit movement, or in the case of angular control movement, unit radian throw. For a  $10^\circ$  control movement a ratio is set up which establishes the potentiometer setting for a  $10^\circ$  throw. A lesser or greater movement would, of course, be represented by a proportionately greater or lesser potentiometer setting. An example is calculated below to obtain the potentiometer setting representing  $M_{\delta_e}$  for a  $10^\circ$  control movement.

$$M_{\delta_e} = 76.8, \text{ REPRESENTS THE EFFECT CAUSED BY A } \delta_e \text{ OF 1 RADIAN.}$$

$$1 \text{ MACHINE UNIT} = 100 \text{ VOLTS}$$

∴ THE  $M_{\delta_e}$  POTENTIOMETER SETTING IS

$$\frac{76.8}{100} \text{ VOLTS OR } .768$$



That is to say that when one machine unit, or 100 volts, is placed across the  $M_{\delta_e}$  potentiometer set at .768, the analogue of the change to the pitching moment caused by an elevator throw of one radian will enter the equations of motion as a forcing function. Ten degrees would be this value multiplied by the fraction  $\frac{10^\circ}{57.3^\circ}$  or .134. The other potentiometer settings representing the change caused by control movements are similarly computed. For the basic airplane these values appear in Tables VI and VII.

The computer diagram for the simultaneous solution of these six non-linear equations of motion is presented as Fig. 9. The inertial coupling terms, which are the product or moment of inertia factors and velocity terms such as  $q \frac{(I_z - I_y)}{I_x}$ , are handled by the servo-multiplier units as indicated in Fig. 9.

A check was made on the analogue computer solutions obtained for the basic airplane (Case I) at sea level using equations (III-28) of Ref. 1 for the longitudinal motion and equations (III-97) for the lateral motion.

III-28, Ref. 1

$$\begin{aligned}\omega_{m_{sp}} &= \sqrt{M_q Z_w - U_o M_w} \\ \xi_{sp} &= \frac{-1}{2\omega_{m_{sp}}} \sqrt{g(M_w Z_u - M_u Z_w)} \\ \xi_p &= -\frac{X_u}{2\omega_{m_p}} - \frac{\xi_{sp} \omega_{m_p}}{\omega_{m_{sp}}} - \frac{M_u(U_o X_w - g) - X_w Z_u M_q}{2\omega_{m_p} \omega_{m_{sp}}^2}\end{aligned}$$







Calculation:

$$\omega_{MSP} = \sqrt{(-16.86)(-2.70) - (+670)(-.034)}$$

$$= 8.27 \text{ rad/sec}$$

$$\zeta_{SP} = -\frac{1}{2 \times 8.27} [670 \times 0 + (-2.70) + (-16.86)]$$

$$= 1.184, \text{ MORE THAN CRITICALLY DAMPED}$$

$$\omega_{MP} = \frac{1}{8.27} \sqrt{32.2 [(-.034)(-.130) - 0]}$$

$$\omega_{MP} = .0456 \text{ rad/sec.}$$

$$\zeta_P = -\frac{(-.015)}{2(.0456)} - \frac{(+1.184)(.0456)}{+8.27} - \frac{0 - (+.0269)(-.130)(-16.86)}{2(.0456)(8.27)^2}$$

$$\zeta_P = +.1691 ; \quad \zeta_P = .17 \text{ MEASURED USING FIG. III-28, REF. 1}$$

$$\omega_P = \sqrt{1 - \zeta_P^2} = \sqrt{1 - .0286} = .045 \text{ rad/sec.}$$

$$T_P = 139.6 \text{ sec. ; } T_P (\text{MEASURED}) = 139.5 \text{ sec.}$$

III-97, REF. 1)

$$\frac{1}{T_s} = \frac{g/U_0 (N_\beta L_R - L_\beta N_R)}{Y_r L_P N_R + L_P N_\beta + \left(\frac{g}{U_0}\right) L_\beta}$$

$$\frac{1}{T_r} = \frac{Y_r L_P N_R + L_P N_\beta + \left(\frac{g}{U_0}\right) L_\beta}{-N_\beta}$$

$$\omega_{MD} = \sqrt{N_\beta}$$

$$\zeta_D = \frac{-Y_r - L_P - N_R - \frac{1}{T_r} - \frac{1}{T_s}}{2\sqrt{N_\beta}}$$

$$\omega_D = \omega_{MD} \sqrt{1 - \zeta_D^2}$$

Calculations:

$$\frac{1}{T_s} = \frac{32.2}{670} [(+.0125)(+.1785) - (-.054)(-.190)]$$

$$(-.173)(-.653)(-.190) + (-6.53)(+.0125)(670) + \frac{32.2}{670} (-.054)(670)$$

$$\frac{1}{T_s} = +.00457 \frac{1}{\text{sec}}$$



$$\frac{1}{T_R} = \frac{-56.655}{-(+.0125)(670)}$$

$$\frac{1}{T_R} = +6.77 \frac{1}{\text{sec}}$$

$$\omega_{M_D} = \sqrt{(670)(+.0125)}$$

$$= 2.89 \text{ rad/sec.}$$

$$\xi_D = \frac{-(-.173) - (-.653) - (-.190) - (6.77) - (.00457)}{2 \times 2.89}$$

$$= +.0208 ; \xi_D(\text{MEASURED}) = .037$$

$$\omega_D = \omega_{M_D} \sqrt{1 - \xi_D^2}$$

$$= 2.88 \text{ rad/sec.}$$

$$\frac{\omega_D}{2\pi} = N_D ; \frac{1}{N_D} = T_D$$

$$T_D = \frac{2\pi}{2.88} = 2.18 \text{ sec.} ; T_D = 2.23 \text{ sec measured.}$$

SINCE THE CALCULATED VALUES CHECK THE RESULTS OBTAINED FROM THE REAC SOLUTIONS, THE REAC SET UP WAS ASSUMED TO BE CORRECT AND THE INVESTIGATION PROCEEDED.

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An investigation into some  
of the effects of cross coupling  
between the longitudinal and  
lateral equations of motion of  
a fighter type aircraft at  
subsonic speed.

